THE STRENGTH OF THE EARTH'S CRUST

JOSEPH BARRELL

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I. GEOLOGIC TESTS OF THE LIMITS OF STRENGTH

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Preface

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PREFACE

The publication of a series of papers on "Diastrophism and the Formative Processes" by T. C. Chamberlin was begun in the *Journal of Geology* in October, 1913. The second part, on "Shelf-Seas and Certain Limitations of Diastrophism," is nearly identical in substance with a portion of a paper read by Professor Chamberlin on August 13, 1913, at the Twelfth International Geological Convention at Toronto, Canada. In this part particularly it is pointed out that the parallel surface and bottom of the shelf-seas, also their occasional extension as shallow water bodies over considerable portions of the continent at certain times, indicate a relation to sea-level and wave base rather than to a delicate isostatic adjustment. The implications of this and other lines of argument given by Chamberlin are toward crustal rigidity, not crustal mobility.

The first four parts of the present article on "The Strength of the Earth's Crust" had been completed before the writer read Professor Chamberlin's paper, or knew that he was at work upon the subject; but the conclusions are so closely in accord with his, though reached by other lines of attack, that this article may be

regarded as a continuation of the same subject, an added contribution in the large field of diastrophism and the formative processes, following out certain of its ramifications.

A somewhat general survey is given here of the problem of the strength of the crust, beginning with the lines of evidence which bear upon it and following out to some degree the conclusions drawn from it. It has in this way been cast into the form of those articles published by the *Journal of Geology* from time to time, under the caption of "Studies for Students."

PART I. GEOLOGIC TESTS OF THE LIMITS OF STRENGTH

INTRODUCTION AND SUMMARY

The capacity of the outer crust to resist vertical stresses is an important field in the theory of dynamical and structural geology. On the one hand, it is known that the larger segments, those of continental and oceanic proportions, rest to a large degree in isostatic equilibrium, the subcrust of the continental areas being lighter than that of the oceanic areas in proportion to the regional elevation. On the other hand, the minor features, those which enter into the composition of the landscape, are known to have been sculptured by external forces and are to be explained therefore as sustained by reason of the rigidity of the crust.

Between these two extremes in magnitude of terrestrial relief lie mountain ranges, plateaus, and basins; made in part by tangential forces, modified by erosion and sedimentation. To what extent can these constructional and destructional forces work in opposition to those other forces which by producing vertical movement make for isostatic equilibrium? The method of attack is from two directions. The geologist examines the structures imposed by tangential forces, the mountains built by igneous extrusion, the surfaces made by erosion, the strata consequent upon sedimentation. From them he may determine the amount of strain which the crust can endure before periodic movements occur in the direction of relief from strain. The geodesist, by means of the plumbline and pendulum, determines the subcrustal densities and notes the degree to which these are balanced against the relief, pointing

therefore to a relation of flotational equilibrium within the solid earth.

Most geologists in former years have utilized but little the principles of isostasy, as may be seen by reference to the standard manuals. On the one hand, the weight of sediments may be spoken of as the *cause* of downsinking with such equal pace that the condition of a shallow sea prevails for a geologic period, though perhaps accompanied by the deposition of thousands of feet of sediment. On the other hand, and without argumentation to explain the apparent inconsistency, the same geologist may state that tangential forces have built folded mountains miles in height which may be subsequently largely removed by erosion before marked vertical warping of the crust occurs.

In contrast to the geologists, certain geodesists have argued in recent years for a high degree of isostatic adjustment; isostasy being regarded by Hayford, for example, as largely complete in areas probably between one square mile and one square degree in size, the mean departure of these unit areas from the level of complete compensation being stated by him as ranging from 250 to 570 ft. These figures he does not regard, however, as of a high order of accuracy, the latter being probably the more reliable of the two. He states that their significance is mainly in showing that isostatic compensation is nearly perfect. It has even been argued by Dutton, Willis, and Hayford, as an outflow of geodetic studies, that those vertical movements of the outer crust which tend to give isostatic equilibrium are the ultimate causes of the periodic great compressive movements.

There is here between geologists and geodesists a tendency to a fundamental difference of opinion, resulting from the emphasis upon one or the other of those opposing forces which work in the outer crust. The truth must lie within the broad zone between these two extremes of theory. To try to bring them together in harmony is the problem before us.

The first part of the paper, on the geologic tests of the limits of strength, opens with a brief review of the lines of geologic evidence which may be used as tests of the degree of resistance or response by the crust to vertical stresses, having regard to both area and

intensity. Deltas built into deep seas seem best adapted to give quantitative measurements. Those of the Nile and the Niger therefore are subjected to detailed study. They indicate that the earth is competent over those regions to sustain stresses due to sedimentation which are measured by the weight of several thousand feet of rock, even where the load is continuous over tens of thousands of square miles. Whatever response there may be is so slow that the deposition is able to keep pace with subsidence and maintain the load as a permanent stress of this magnitude upon the crust. By analogy the conclusion may be applied to other parts of the earth, and to those negative loads created by the erosion to base-level of regions previously unwarped to an elevation presumably near to that which would give isostatic equilibrium. Consequently, also, the crust should be able to bear in considerable degree the folded and overthrusted structures piled up by the tangentially compressive forces which periodically operate to such large degree within its outer shell. Deeper changes, involving changes of density, are involved, however, in orogenic processes and express themselves in vertical warpings associated with, and following after, folding. This association of vertical and tangential forces complicates the problem of the crustal strength needed to support mountain ranges.

The measures derived from the study of deltas are more in accord with those larger estimates of the strength of the crust obtained by Putnam and Gilbert in 1895 from a transcontinental series of gravity measurements in which was developed and employed for the first time the conception of local rigidity but regional isostasy.[1] Their conclusions have been thought to be superseded and controverted, however, by much more elaborate and complete geodetic studies, first by Hayford, and later by Hayford and Bowie, which went to show that the crust was very much weaker and in much more perfect static equilibrium.

The calculations of Hoskins tended to show also that the crust within the zone of isostatic compensation could not bear permanently loads as great as those apparently imposed by these deltas. If, however, the great hydrostatic pressures within the deeper crust

[1] *Bull. Phil. Soc. Wash.*, XIII (1895), 31–75; *Jour. Geol.*, III (1895), 331–34.

give to it an added resistance to stress differences as great as indicated by the experiments of Adams, then the strains imposed by the deltas may be permanently borne.

This confrontation of the conclusions drawn from various paths of approach raises the problems which are treated in the second part.

MOUNTAINS BUILT BY COMPRESSION OR IGNEOUS ACTIVITY

Mountain ranges made by folding or extravasation must be independent to some degree from vertical forces, but these are not suitable geologic tests of the rigidity of the crust, since it is known, as noted in the introduction, that they are secondarily connected with diminutions of density in the zone of isostatic compensation and in many cases are rejuvenated after partial erosion by later upwarping.

The individual mountains or plateau remnants left standing by circumdenudation, or piled up as volcanic cones are clearly burdens upon the earth. The volume which rises above the average level is a measure of the stress. Gilbert has so used them and obtained values ranging from 40 to 700 cubic miles.[1] These volumes, however, might be called minimum estimates, as may be seen upon examination of their nature.

If a certain broad upwarping reduces the vertical stresses to a minimum and erosion follows without further adjustment, it is the volume of the valleys rather than the mountains which soon comes to measure the larger possible departures from equilibrium. The remaining mountains by their weight produce local downward stresses, but the more regional stresses are upward and are due to the breadth of the field of erosion. These regional stresses will become larger ultimately than the local stresses due to the residual masses.

Volcanic cones do not continue to be built up until their base begins to sink into the crust as fast as the upward growth takes place. On the contrary, their growth ceases when the hydrostatic pressure of the high column of lava or a decadence of pressure in the reservoir below leads finally to a shifting of the vents.

[1] "The Strength of the Earth's Crust," *Bull. Geol. Soc. Am.* (1889), I, 25.

Regional igneous activity has poured out lavas and breccias, burying previous mountainous topography and adding thousands of feet to the outer crust. Lack of simultaneous erosion, as in the Miocene flows of the Columbia plateau, shows that subsidence progressed, perhaps with approximately equal pace. The present altitude of the Columbia plateau is youthful, as shown by the steep canyon walls and undissected interfluvial areas. The initial subsidence accompanying igneous outpouring and the distinctly later upwarping without compression suggest that here isostasy has prevailed. But in such regions the geologic evidence points toward a minimum strength of the crust. The wide area of activity, the numerous vents, the general absence of localization, all are suggestive of widespread fluid rock beneath, magmas which are probably far above the level where the accompanying temperatures are normal. Such conditions would seem to imply the impossibility of the outer crust carrying over such regions the stresses which are possible in regions long free from igneous activity. More reliance as maximum measures of the strength of the crust should be placed therefore upon those external changes which are entirely independent in origin from the interior of the earth locally beneath them.

SHIFTINGS OF LOAD DUE TO CLIMATIC CHANGE

Some of the most striking examples of loading and unloading of the crust are those connected with the climatic fluctuations of the Pleistocene. The continental ice sheet formed, advanced, and retreated rather rapidly, as viewed from the geologic standpoint. As it retreated, the lacustrine and estuarine shores show that the land was rising with the melting of the ice. The upwarping accompanying deglaciation was limited to the approximate region of maximum glaciation and was greatest in the direction where the ice was thickest, in the St. Lawrence valley the maximum uplift being more than 600 ft. These relations suggest strongly an isostatic response to the relief of load. It is not known, however, to what degree the previous downwarp compensated for the burden of the continental ice sheet and what degree of regional stress the crust was able to bear. The lack of close response is seen in that the upwarp continued as a residual movement after the ice departed.

The movement of the crust could not keep pace with the climatic change but it shows by means of these fossil water planes its incompetency to bear without at least partial yielding a burden as broad and as heavy as the Pleistocene climates placed upon it.

Gilbert, in 1889, was led by reflection upon the changes of load imposed by the waters of extinct Lake Bonneville to use them as a measure of the strength of the earth's crust to resist isostatic adjustments,[1] and as previously stated, tested the conclusions drawn therefrom by comparisons with the volumes of mountains made by extravasation, or circumdenudation, or their combination, and of valleys of erosion. Of Lake Bonneville he states:

> Considering the main body of Lake Bonneville, it appears from a study of the shorelines that the removal of the water was accompanied, or accompanied and followed, by the uprising of the central part of the basin. The coincidence of the phenomena may have been fortuitous, or the unloading may have been the cause of the uprising. Postulating the causal relation, and assuming that isostatic equilibrium, disturbed by the removal of the water, was restored by viscous flow of crust matter, then it appears (from observational data) that the flow was not quantitatively sufficient to satisfy the stresses created by the unloading. A stress residium was left to be taken up by rigidity, and the measure of this residuum is equivalent to the weight of from 400 to 600 cubic miles of rock.
>
> From these phenomena and theoretic considerations arises the working hypothesis that the measure of the strength of the crust is a prominence or a concavity about 600 cubic miles in volume.

THE EVIDENCE FROM EROSION CYCLES

Erosion base-levels folded and uplifted tracts, leaving for a time during the process mountains of circumdenudation whose local stresses have previously been discussed. The development of peneplains implies a rigidity of the crust sufficient to prevent responsive vertical movement until after the completion of the cycle of denudation. It may be difficult to determine the original average elevation and the degree of progressive uplift *pari passu* with erosion which preceded the peneplanation, but the fact that broad areas become flat and are controlled until the next deformative movement by the level of the sea suggests that they cannot

[1] *Bull. Geol. Soc. Am.*, I (1889), 23–27.

lie after erosion in close isostatic equilibrium; that whatever stress this implies can be carried by the earth for long periods of time.

The ancient peneplains are now broadly warped and uplifted. The rivers, as a rule, are intrenched in youthful valleys; or their seaward courses are drowned and not yet reclaimed by delta building. These features testify to the recency of world-wide crustal unrest, marked chiefly by movements of a vertical nature; movements which presumably diminished the vertical stresses in the outer portions of the earth and has produced at the present time, as Willis has argued, a higher degree of isostatic compensation than has been customary through the long periods of quiet which separate the epochs of movement.

There are difficulties, however, in using ancient base-leveled surfaces now upwarped as measures of the previous stress. It is known that a region like the Colorado plateaus which now stand markedly high tended to lie near sea-level from the beginning of the Paleozoic to the end of the Mesozoic. Presumably a decrease of density within the zone of isostatic compensation has taken place here during the Cenozoic and the uplift has accompanied or followed the internal change.

Furthermore, if there are stages in the uplift, a considerable volume of rock is removed during each stage, so that at no one time has the average elevation of the region been as high as the residual masses might be thought to imply. Allowing for these qualifications, however, there seems no doubt that the study of erosion cycles will throw light upon the limits of stress due to unloading which the crust can resist, and also upon progressive changes in subcrustal densities through geologic times. This evidence of considerable crusted rigidity, shown by freedom from compensating movements during a cycle of erosion, or by warpings not in sympathy with isostatic stresses during cycles of crust movements, has been pointed out before. Hayford has sought to explain it away by invoking, first, the slight crustal cooling which would occur in regions of erosion because of removal of the upper rock, heating in regions of deposition. Second, he assumes as probable the existence of a high coefficient of compressibility sufficient to make eroded regions rise in appreciable ratio to the thickness of

the load eroded. Third, he assumes a crustal undertow from heavy toward high areas which would not only fold the surface rocks and heat them in the region of undertow but restore the equilibrium of mass in the regions of erosion and deposition.[1] It may be said of all of these factors that when they are subjected to quantitative statement they appear so trifling as to fail wholly to explain the magnitude and breadth and periodicity of crust movements. The inadequacy of the temperature effects has been pointed out clearly by Harmon Lewis.[2] The assumption of the high coefficient of compressibility involves more instead of less difficulty for the high isostasist. The inadequacy of isostatic undertow to account for folding has been discussed briefly by the present writer elsewhere.[3] On the other hand, the control of the level of the earth's surface during epochs of quiet by the forces of planation and not by forces making for close isostatic adjustment has been discussed convincingly by Chamberlin in his present series of articles. It seems clear, then, that in the study of cycles of erosion and deposition much may be determined in regard to the limits of terrestrial rigidity. The subject could be developed further, but it is preferred to place the emphasis of this paper upon the more readily estimated loads produced by the building of deltas.

THE EVIDENCE FROM DEPOSITION

Preliminary statement.—The waters deposit sediment upon the depressed areas of the crust. To what extent may such areas be loaded before yielding of the base and resultant subsidence take place? The geologic record makes it clear that subsidence and deposition are necessarily related. It has been stated often that deposition was the cause and subsidence the effect, the two being regarded as in delicate isostatic adjustment. But this is in reality an assumption, for such a supposed relationship overlooks the extent to which subsidence might have gone forward without deposition and ignores the external load which may have been necessary to

[1] "The Relations of Isostasy to Geodesy, Geophysics, and Geology," *Science*, N.S., XXXIII (1911), 199–208.

[2] "The Theory of Isostasy," *Jour. Geol.*, XIX (1911), 622, 623.

[3] Joseph Barrell, *Science*, N.S., XXIX (1909), 259, 260.

perpetuate and add to a crust movement initiated by internal causes. Sedimentation is dependent upon the rate and continuity of subsidence as well as upon the rate of deposition. Thus, although the sediments give the most complete record of crustal movements, for the distant past it is not easy to separate cause and effect and ascribe to each its part. Where the thickness of sediments, however, is small, as over much of the continental interior, the cause of submergence is presumably almost wholly independent of the local load. Where the sediments are thick and subsidence rapid, as within the geosynclines, the load imposed by sedimentation may on the contrary become the controlling force. It is a particular phase of deposition, however, which will be considered in this article, a study of the load imposed upon the crust by certain deltas. As long as the water plane lies at a constant level the delta builds out at its front. Upon subsidence of the supporting crust the shore retreats inland; less sediment reaches the now submerged front, and the delta in consequence grows chiefly by additions to the shoreward part of its upper surface. The two methods of growth not uncommonly alternate upon the same delta, showing the discontinuity of subsidence. In building outward a delta acquires a convex shoreline. This form is clearly related to aggradation, not to isostatic uplift, and its volume is a measure of a load inclined to further sinking, the larger rivers tending to drain toward and into the downwarps of a continent. To what degree, then, can a region of the crust which is possibly already resisting downward strain bear this added burden? A preliminary examination will be made of several classes of deltas in order to choose those best adapted to test this question.

Most of the deltas of Eurasia and South America are at present advancing rapidly into shallow embayments and the faunas of the continental islands show that the latter were recently a part of the land. The physical and organic evidence thus concur in showing that a very recent subsidence has taken place. It is to be concluded that a submergent phase in the Cenozoic crustal oscillations has marked the short interval since the last retreat of the Pleistocene ice. The great deltas constructed during the late Tertiary and in the Pleistocene are consequently now in great part drowned.

Their location, volume, and limits in most cases are not known. Their modern and smaller representatives, as they build out into shallow water, do not greatly increase the load upon the crust. Deltas recently drowned are therefore not well adapted to serve as tests of the strength of the crust.

Deltas which lie in re-entrant angles of the continents are also poorly adapted to be used as a test. Those of the Indus, the Ganges, and the Colorado are illustrations. As they fill up the heads of gulfs and are without the typical convex outline, it is not only difficult to compute their volume but their situation is such as to suggest that even without the construction of the delta the region might be far out of isostatic adjustment.

Certain rivers, which face the open ocean, such as the Columbia, do not build deltas because of the power of the waves and currents which sweep laterally the fine detritus.

Many rivers, however, build considerable submarine deltas even where the in-planing forces of the ocean prevent a terrestrial outward growth. Such submarine deltas, owing probably to the power of the waves rather than to recent submergence, are marked by convexities in the bathymetric contours opposite the river mouths. The Congo, the Orange, and the Zambesi are examples. These hidden deltas which are built out into deep waters cannot reach more than a certain distance from the shore and part of their detritus is carried laterally along shore by the waves, but nevertheless they possess a very considerable volume and the convexity which they make upon the ocean floor shows to that degree the rigidity of the crust.

The maximum test is found where great rivers have carried forward subaerial topset beds of their deltas over what was previously deep ocean. Fluviatile construction in such examples has dominated over marine destruction, giving a convex outline to the shore; but the subaqueous deposits may still make up the greater part of the volume. Even in these cases the question may be raised whether the deltas have attained the maximum possible size permitted by the strength of the crust. Their size may, on the contrary, be limited even here by the balance of the surface agencies and the limited time during which the river has dominated over

the sea. It is a fair presumption, however, that the largest deltas have reached a size where subsidence keeps pace with added volume.

The deltas of the Nile and Niger.—Only the most powerful rivers, laden with abundant waste and protected by their situation from the heavier wave and current action, can build deltas of this last class directly into ocean basins. Perhaps the two best of the few good examples are those of the Nile and the Niger. Both have built out great deltas from regularly curving shores of the Atlantic type—the type where recent folded mountains do not mark the line between continent and ocean, the type where tangential forces

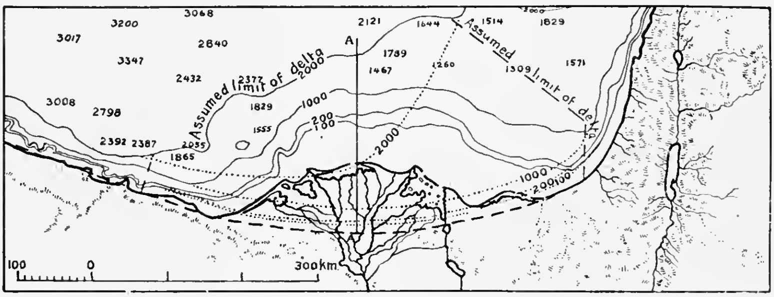

FIG. 1.—Delta of the Nile. Scale 1: 10,000,000. From Andree's *Allgemeiner Handatlas*, vierte Auflage.

cannot be supposed to have disturbed recently the isostatic balance of continent and ocean.

To determine the areas, depths, and volumes of the deltas from the standpoint of isostasy, a smooth curve, as shown in Figs. 1 and 3, was continued through them from the shore beyond. The submarine contours were also projected in dotted lines, giving the form of the bottom as it presumably would now be if no rivers at these places had entered the sea. The volume of the deltas may then be determined by computing the volume included between these two sets of contour lines.

In both cases, in so far as the positions of the hypothetical bottom contours are open to doubt, they have been located somewhat above a most probable position, so as to tend to throw the error of computation in the direction of too small rather than too

large a volume. For instance, the easterly drift of the water facing the Nile delta may have carried considerable mud in suspension to beyond the line assumed here as its limits. In consequence, the hypothetical 2,000-meter contour should be drawn perhaps much closer to the coast of Palestine than has been done. Beneath the Niger delta the contours lie close together on the west but have been drawn as spreading apart toward the east. It would perhaps be nearer the truth to project the steep character of the coastal slopes to the east of the Niger delta under it to where the contours meet the chain of volcanic island mountains extending from the Cameroons out to sea. This appears to be especially probable, since Buchanan has shown that the gentle slopes of the Guinea coast even beyond the limits of the deltas, and extending from

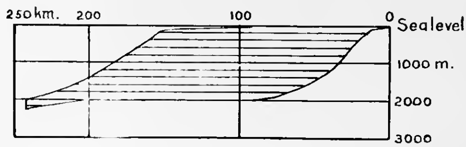

FIG. 2.—Vertical section of the delta of the Nile on *A-A*, Fig. 1. Horizontal scale 1: 5,000,000. Vertical scale 1:200,000. Area of section, 295 kilometers.

long. 2°30′ E. to lat. 8° S., are mantled throughout by very soft, black, oozy mud, characteristic of river estuaries.

All the way down the coast as far as Loanda, lat. 8° S., the same gentle gradients and the same very soft river mud were found. It appears that the land débris brought down by the Niger and Congo, and by other less important rivers, is collected and concentrated in this district. The prevailing current past the mouth of the Congo is a northerly one, while all along the coast from Cape Palmas to the Niger an easterly current sets. These help to confine the drainage matter of both rivers to a comparatively small extent of littoral. If from the soundings west of Cape St. Paul we compute the mean continental slope, we find that the 500-fathom line is at a mean distance of 4.1 miles, the 1,000-fathom line at 11.7 miles, and the 1,500-fathom line at a distance of 17 miles from the 100-fathom line. If it is assumed that in the absence of the Niger and the Congo the continental slope would be much the same as the average found in the profiles west of Cape St. Paul, it may be concluded that the excess of mud forming the flatter talus along the coasts affected by these rivers is due to the mud brought down by them.[1]

[1] J. Y. Buchanan, "On the Land Slopes Separating Continents and Ocean Basins, Especially Those on the West Coast of Africa," *Scottish Geographical Magazine*, May, 1887, pp. 7, 8.

Buchanan states that this gentle bottom slope extends for 1,100 miles along the coast, and computes the volume contained between the steep gradient presumably once existing and the flatter gradient of the present bottom. This represents a deposit of 66,-000 cubic nautical miles of detritus due principally to the Niger and the Congo.[1] This great volume cannot be used safely, however, as the measure of a load upon the crust, since a believer in the theory of close isostatic compensation could claim with some degree

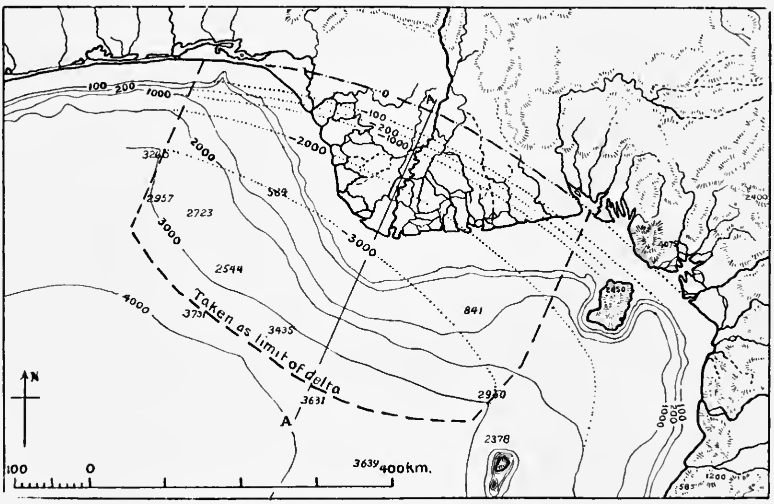

FIG. 3.—Delta of the Niger. Scale 1:10,000,000. From Andree's *Allgemeiner Handatlas*, vierte Auflage.

of reason that the initial slope of the concave shores of the Gulf of Guinea need not have been as steep as the bold convexity of Africa to the west, or that the load may have depressed the bottom so as to have equalized the pressures. Furthermore, Buchanan does not include any of the land area of the Niger delta. The following estimates will give the volume only of the clearly constructional part of the Niger delta, including both the land and

[1] *Op. cit.*, p. 8 and Fig. 3. The volume stated by Buchanan appears to be correct if the two profiles have a common point taken upon the *shoreline*. In his figure, however, the common point A is shown as upon the 100-fathom contour. From this error in the diagram given by Buchanan the volume estimated from the diagram would be much less than 66,000 cubic nautical miles.

the sea portion. But it will be seen, from Buchanan's statements, that this is a minimum estimate of the areal load imposed by the rivers, for a more or less continuous burden on the crust would appear to stretch for a thousand miles along this African coast, reaching a maximum unit value, however, in the great delta of the Niger.

The outer limits of the deltas were taken where the convex slopes fade out into the general ocean bottom.

The results of computing the volumes shown between the two sets of contour lines are as follows:

TABLE I

Delta of the Nile

Area within 1,000-m. contour	71,000 sq. km. (27,400 sq. mi.)
Area within 2,000-m. contour	106,000 sq. km. (38,800 sq. mi.)
Radius of equivalent circle	175 km. (110 mi.)
Equivalence in equatorial square degrees	8.6 sq. degr.
Average thickness within assumed limits	0.84 km. (2,800 ft.)
Equivalence in rock upon land	0.46 km. (1,540 ft.)
Ratio to 76 miles of crust	1 to 260=0.0038
Maximum thickness	2.0–2.3 km. (6,600–7,600 ft.)
Equivalence in rock upon land	1.1–1.3 km. (3,600–4,200 ft.)
Volume within assumed limits (extending on the east to somewhat below 2,000 m.)	89,000 cu. km. (21,300 cu. mi)
Equivalence in rock upon land	50,000 cu. km. (11,700 cu. mi.)

TABLE II

Delta of the Niger

Area within the assumed limits	195,000 sq. km. (75,300 sq. mi.)
Radius of equivalent circle	250 km. (155 mi.)
Equivalence in equatorial square degrees	15.8 sq. degr.
Average thickness within assumed limits	1.1 km. (3,600 ft.)
Equivalence in rock upon land	0.6 km. (1,980 ft.)
Ratio to 76 miles of crust	1 to 200=.005
Maximum thickness	3.0 km. (9,900 ft.)
Equivalence in rock upon land	1.65 km. (5,450 ft.)
Volume within assumed limits	217,000 cu. km. (52,000 cu. mi.)
Equivalence in rock upon land	120,000 cu. km. (27,000 cu. mi.)

The deltas in their growth had displaced their volume of water. The added loads which they throw upon the crust are measured by

subtracting the weight of the water from that of the sediments. A specific gavity of 2.67 has been taken by geodesists as the average for the outer shell of the earth. The degree of consolidation of the deeper parts of the deltas is not known, but for present purposes the specific gravity of their sediments as a whole may be assumed as 2.50. This will be near the truth if the composition is that of the average shale, if 10 per cent of pore space be assumed and this is wholly filled with water. The specific gravity of sea water is 1.03, leaving an effective specific gravity for the sediments of 1.47. The ratio of 1.47 to 2.67 is 0.55. The thicknesses given for the deltas should therefore be multipled by this factor for estimating the equivalent burdens of rock of specific gravity of 2.67 above sea-level.

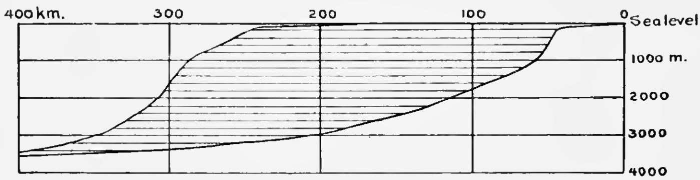

FIG. 4.—Vertical section of the delta of the Niger on *A–A*, Fig. 3. Horizontal scale 1:5,000,000. Vertical scale 1:200,000. Area of the section, 645 kilometers.

It is seen that the deltas are in the form of inclined double convex lenses. Thicknesses approaching the maximum are found over considerable areas in the middle. The load imposed by this thickness is equivalent in the Nile delta to 3,600–4,200 ft. of rock above sea-level; in the Niger delta to 5,000–5,500 ft.

Discussion of results.—The region of the southeastern Mediterranean is held by Suess to be geologically of very recent origin, downfaulted from the continent. The delta of the Nile, much smaller than that of the Niger, is therefore to be regarded as young and may be still increasing in volume.

The great size of the Niger delta suggests, on the other hand, that it may have reached the limit permitted by the strength of the crust. Subsidence may now intermittently keep pace with deposit. If the 1,000-meter contour has been located correctly, as shown in Fig. 3, it suggests that such may be the case, since it is

seen that in contrast to the Nile delta the slopes are much steeper between the 1,000- to 2,000-meter than between the 200- to 1,000-meter contours. This can be explained by assuming that the steep slope lying below and beyond a flatter slope was once a foreset slope just below wave base, whereas it now lies at least 800 meters below. If such a subsidence has occurred, it appears, however, to have been confined to within the limits of the delta; since a peripheral overdeepening of the ocean floor is not evident. On the other hand, it is noted by Penck, but probably too sweepingly, that all bathymetric curves have their steepest slopes between 1,000 and 2,000 meters in depth.[1] Such a phenomenon might be due to lateral flow of sediment under a certain depth of load and without relation to subsidence of the base. The question whether the load of the Niger delta is as great as the crust can bear is therefore an open one.

The Gulf of Guinea, where now the delta is built, is regarded by many geologists as having originated since the Middle Mesozoic by a breaking-down from the continent of Gondwana, but the presence of Middle Cretaceous marine beds skirting much of the coast of West Africa suggests perhaps that the delta in its construction does not go back of the Tertiary. In fact it would seem possible from the youthful relief of the continental plateau that the delta built from its waste is of Upper Tertiary and Pleistocene growth.

A single delta might happen to be a mere veneer of sediment upon an originally slightly submerged projecting part of the coast. Such a fortuitous coincidence of unrelated circumstances may, however, be dismissed as highly improbable in the case of two great rivers draining in opposite directions from the same continent. The conclusion that these deltas are really externally constructive features and measure a real strain upon the crust is strengthened by noting the submarine deltas opposite the other great rivers of Africa, built into the ocean, even though the waves and currents have limited them by preventing their subaerial seaward growth.

In the mechanics of the relation of the delta to the stresses in the crust an important factor is the nature of the marginal land. Shores of the Pacific type have great mountain systems marginal

[1] *Morphologie der Erdoberfläche*, I (1894), 146.

to the continents. Parallel to them the sea has great fore-deeps. It appears as though the mountain ranges had been piled too high by tangential forces, and, by virtue of the partial rigidity of the crust, had depressed the neighboring ocean bottoms. Erosion of the coastal mountains and deposition of their waste in the fore-deep would tend, up to a certain limit, to equalize the strain in the crust. In that case it might happen that, although the mass of the delta measures a stress, this might be opposite in character to pre-existing stresses, with the result that the strain upon the crust beneath the delta before the infilling might be as great or greater, but in an opposite direction. The greatest remaining strain within the sea-bottom could conceivably be an upward strain under the parts of the fore-deep not filled.

Such relations are not found around abyssal slopes of the Atlantic type. These are regarded by many geologists following the lead of Suess as made by marginal downbreaking of the continents. They have but little or no relation to the older folded structures and no excessive deeps parallel to the continental margins. If these relations of the Atlantic and Indian oceans to the continents are rightly interpreted as to cause, it is probable that the stresses which make for downsinking extend beyond the parts already foundered. The margin of continents and ocean basins are not likely to be depressed too low, but if remaining out of isostatic adjustment they would tend rather to stand too high. There is no theoretic reason to believe, therefore, that the Nile and Niger deltas have neutralized pre-existing strains. They are best regarded as real and present burdens sustained by the rigidity of the crust.

Whether or not, however, the building of deltas produced stresses of a character identical with, or opposite to, those previously existing in the region, the stress gradient between the areas of the delta and the surrounding areas would be measured by the weight of the sediments, and this would tend to produce differential flexure. It would seem to be a logical conclusion, therefore, from these tests, that certain parts of the earth's outer crust can resist for considerable periods of time vertical stresses at least equivalent to the weight in air of 10,000–25,000 cubic miles of rock in lenslike

forms spread over areas of 40,000–75,000 square miles and reaching thicknesses in air over considerable areas of 4,000–5,000 feet.

The tabulation of the data regarding the deltas shows the area of the Niger delta to be equivalent to a circle 310 miles in diameter and that over this area the load of the delta is one two-hundredths the weight of the crust to a depth of 76 miles, this being the depth of the zone of isostatic compensation given by the latest determination of Hayford.

According to Hoskins, in a calculation made for Chamberlin and Salisbury,[1]

a dome corresponding perfectly to the sphericity of the earth and formed of firm crystalline rock of the high crushing strength of 25,000 pounds to the square inch, and having a weight of 180 pounds to the cubic foot, would, if unsupported below, sustain only $\frac{1}{525}$ of its own weight. This result is essentially independent of the extent of the dome, and also its thickness, provided the former is continental and the latter does not exceed a small fraction of the earth's radius.

The delta, though large, is so limited in size in comparison with continental areas that it would be somewhat more effectively supported, but its externally convex form can hardly be supposed to give it added domal strength, since it consists of more or less unconsolidated material piled upon a concave floor.

The theory of isostasy holds that at a certain depth in the crust there is an approach to equal pressures, the larger relief of the surface being balanced in large part by subsurface variations in density. The larger segments of the crust tend to rise or sink until the elevations are in adjustment to the density beneath. A corollary of this theory is that unbalanced surface loads are largely sustained by the strength of the crust above this level of equal pressures; in other words, but little of the load is transmitted to the deeper earth below. For purposes of discussion it may then be assumed that the load of the Niger delta is supported by the outer 76 miles of crust. This depth is one-fourth of the diameter of the circle equivalent in area to the delta. The load over this area, as stated, is one two-hundredths of the weight of the supporting crust. Allowing something for the limited area of the delta, it is seen never-

[1] *Geology*, I, 555, 1904.

theless to imply a strength of the crust about twice that assumed as a maximum by Hoskins as a basis for his calculation. There are several contributing factors which may explain the disagreement between the figures obtained by observation of the deltas and the calculation given by Hoskins and others: First, part of the stress is transmitted laterally to some extent into the deeper layers, but as the diameter of the loaded area is four times its depth this can be a partial explanation only and has, furthermore, been allowed for. Second, part of the stress may be transmitted into the deeper earth below the 76-mile zone of isostatic compensation. This is about equivalent to third, that the zone of isostatic compensation may extend deeper, at least locally, and fade out more after the suggestion made by Chamberlin.[1] Fourth, a consideration which the writer regards as most important is that the crust may in reality possess greater crushing strength than the 25,000 pounds per inch postulated by Hoskins. At the time that Hoskins made this calculation it seemed that this figure was the highest which could be chosen, since it is higher in fact than the crushing strength of the average surface rock when subjected for even a short time to compression in a testing machine, and in the earth the stresses must be carried for indefinite periods. The experiments by Adams[2] have shown, however, that under the conditions of cubic compression which exist in the earth the rocks are capable of sustaining for indefinite times far higher stress differences than they could bear even for a short time when subjected to stress in one direction only, as at the surface of the earth. These experiments showed that:

> At ordinary temperatures but under the conditions of hydrostatic pressure or cubic compression which exist within the earth's crust, granite will sustain a load of nearly 100 tons to the square inch, that is to say, a load rather more than seven times as great as that which will crush it at the surface of the earth under the conditions of the usual laboratory test.
>
> Under the conditions of pressure and temperature which are believed to obtain within the earth's crust empty cavities may exist in granite to a depth of at least 11 miles.[3]

[1] *Jour. Geol.*, XV (1907), 76.

[2] "An Experimental Contribution to the Question of the Depth of the Zone of Flow in the Earth's Crust," *Jour. Geol.*, XX (1902), 97–118.

[3] *Op. cit.*, p. 117.

It appears then that, even allowing for the great increase in temperature within the earth's crust at depths greater than can be reached by the limitations of experiment, the demands made upon the strength of the crust by the load of the Niger delta are not greater than can be explained by the theory of the mechanics of materials as now understood. This theory rests, however, even after Adams' experiments, upon only a limited range of laboratory observation, and extending over but limited periods only, thus demanding extrapolation both of stress and of time when applied to the whole thickness of the outer crust and over hundreds of thousands of years. Therefore the study of the direct evidence supplied by geologic observation is more convincing in regard to the limits of crustal strength.

These deltas point toward a measure of crustal rigidity capable of sustaining to a large degree the downward strains due to the piling-up and overthrusting of mountains built by tangential forces, or those resulting from the load of sediments in areas of deposition, or those upward strains produced by the erosion of plateaus previously uplifted toward isostatic equilibrium. A final conclusion must, however, await a further discussion in the later parts.

[*To be continued*]

THE STRENGTH OF THE EARTH'S CRUST

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PART II. REGIONAL DISTRIBUTION OF ISOSTATIC COMPENSATION

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INTRODUCTION AND SUMMARY

The strength of the crust has been tested in the first part of this paper by those geologic changes which alter the surface of the earth, but not the density of its interior. If these changes in load initiate rather than merely coincide with vertical movements which serve to diminish the stress, they are thereby shown to be greater than the earth can permanently endure. If, on the other hand, the constructional forms persist, as in the two great deltas studied, then the movements which may exist in the crust due to those loads must be slower at least than the process of surface construction. Such loads consequently, unless counterbalanced by some factor not apparent, are within the limits of crustal strength.

But surface changes and the loads implied can be measured only in special cases. The previous attitude of the crust and the degree and direction of strain then existing in it are complicating factors which it is difficult quantitatively to evaluate. For these reasons the evidence yielded by geodetic investigation promises, in the end, more general and more accurate results.

It is an important conclusion, established by geodetic evidence, that the ocean basins are underlain by heavier matter than that beneath the continental platforms; the tendency through geologic time for the continents to rise relatively to the oceans may be correlated with this difference in density and the lightening of the land areas by the progressive erosion of the land surfaces. It is believed that the rejuvenative movements are in the direction of isostatic equilibrium. Fortunately for land-dwelling vertebrates, the crust is too weak for readjustment to be deferred until after the erosion of the lands, begun by the subaërial forces, shall have been completed by the sea.

But the power of geodetic research does not cease with the establishment of this cause of the maintenance of the differential relief between land surface and ocean floors. Beneath the surface of the continents it reveals heterogeneities of density and measures them against the more or less local relief above. To the extent to which areas of lighter or denser matter do not correspond to proportionately higher or lower relief, real strains either upward or downward are shown to exist through the crust. Over areas of plains which have not suffered much change for geologic ages, geodesy may thus reveal the existence of large crustal strain. On the contrary, in regions of mountainous relief, although the individual mountains are sustained by rigidity and bring local strains upon the supporting basement, geodetic study may show that there is close regional compensation of density balanced against relief, obliterating with depth the stress differences due to topography. These methods of research are thus capable of attacking the problem of the amount and direction of vertical strain existing in the crust under any part of the land surface and, to a lesser degree of accuracy, the crust beneath the sea. The breadth of the individual areas which depart from equilibrium in one direction may constitute also a vital part of the problem.

But although these are fields of research open to the geodesist, they are cultivated with much labor. The position of many stations on the surface of the earth must be determined by astronomic observations to within a fraction of a second of arc. Then a triangulation network, continent-wide, ties these together and shows

at each station, after allowing for the small errors of observation, what are the deflections of the vertical produced by the variations of relief and density. But this deflection for each station is the net result of all the relief from mean level and all the subsurface departures from the densities necessary to sustain that relief for distances of hundreds and, to a diminishing extent, even thousands of miles. The problem is made more soluble, however, by another and independent mode of attack. Observations on the intensity of gravity, when corrected for latitude, for elevation, for the surrounding relief and the density theoretically needed to sustain that relief, show the vertical component of those outstanding forces whose horizontal component was measured by astronomic determinations. It is seen that if the topography is known and its influence evaluated, and sufficient observations are reduced, the distribution of subcrustal densities and consequently the amount of crustal strains form soluble but complex problems.

The mathematical mode of investigation of such problems has, however, both its advantages and disadvantages. The advantages lie in giving quantitative results and in the test of the accuracy of the trial hypotheses by means of the method of least squares. A disadvantage lies in the necessity of erecting simple hypotheses in place of the complex realities of nature, in order to bring the data within the range of mathematical treatment. The precision of mathematical analysis is furthermore likely to obscure the lack of precision in the basal assumptions and through the apparent finality of its results tends to hide from sight other possibilities of the solution.

It is because of the geologic nature of the hypotheses on which the calculations concerning isostasy rest, and the geologic bearing of the results, that it is no act of presumption for the geologist to enter into this particular field of the geodesist.

The measurements of isostasy have been placed most fully on a quantitative basis by Hayford, and the science of geology is indebted to him in large measure. In the following consideration of the geodetic evidence attention will be confined almost entirely to his work, supplemented by that of Bowie. Hayford was the first to consider the influence of the topography and its compensation

to very great distances from each station, the first to make a considerable number of trial solutions upon various assumptions as to the depth of the zone of isostatic compensation, with the result that the reduction of the observations gave the dimensions of the earth with a considerably smaller probable error than any previous computations.[1]

But the conclusions in regard to the strength of the crust, drawn in the first part of this article from the study of deltas, stand in strong contrast to certain statements by Hayford and later by Hayford and Bowie. This second part must therefore outline the results reached by them and show what reconsiderations are necessary in order to bring into harmony their conclusions and the evidence derived from the previous geologic study. A preliminary review without criticism is given of their work in order to bring out their methods and results, and the geologic conclusions which they draw from those results. It is followed by a re-examination of the subject of regional versus local compensation. This is the problem of the size of the area over which, by virtue of the rigidity of the crust, irregularities of density and topography do not have individual relationships but do largely compensate each other over the region as a whole. It is a measure, therefore, of the areal limits of crustal strength. The tests employed by Hayford and Bowie are, as they note, indeterminate up to radii above 58.8 but less than 166.7 km. in length. Consequently Hayford did not change his opinion, based upon previous investigations, that regional compensation was limited to areas of less than one square degree. In

[1] The final publications have been issued by the United States Coast and Geodetic Survey and are as follows: Hayford, "The Figure of the Earth and Isostasy from Measurements in the United States (up to 1906)," 1909; referred to in this paper as Hayford, 1906; Hayford, "Supplementary Investigation in 1909 of the Figure of the Earth and Isostasy," 1910; referred to in this paper as Hayford, 1909; Hayford and Bowie, "The Effect of Topography and Isostatic Compensation upon the Intensity of Gravity," 1912; referred to in this paper as Hayford and Bowie, 1912; Bowie, "Effect of Topography and Isostatic Compensation upon the Intensity of Gravity" (second paper), 1912; referred to in this paper as Bowie, 1912.

In addition Bowie has published in the *American Journal of Science,* "Some Relations between Gravity Anomalies and the Geologic Formations in the United States," (4) XXXIII (1912), 237–40.

The following discussion of their geodetic measurements and results will be confined to the work in these five papers.

this article, however, two other tests are applied which indicate that although in some areas compensation does not extend to 166.7 km. radius, in other areas it extends farther. It is concluded that the United States shows regional departures from isostasy over areas many times larger than Hayford thought to exist, as broad and in some regions probably somewhat broader than the areas of the Nile and Niger deltas, the breadth depending in considerable part upon the magnitude of the loads per unit of surface.[1]

GEODETIC MEASUREMENTS OF ISOSTASY BY HAYFORD AND BOWIE

Hayford's conclusions from deflections of the vertical.—The positions of many stations over the United States were determined with great accuracy by geodetic measurements from other stations, thus making a closed network. The positions were also determined by astronomic observation. The differences in latitude and longitude between the geodetic and astronomic positions give the observed deflections of the vertical due to the attraction of the surface irregularities and internal heterogeneities of the geoid. To account for these deflections the gravitative attraction upon the plumb-line at each station of all the topography from ocean bottoms to mountain tops within 4,126 km. was computed. The influence of the topography alone upon the direction of the vertical is known as the topographic deflection and averages a little over 30″. The average of the actually observed deflections are, however, but a fraction of this value. Consequently the excesses of volume represented by continents above oceans, and by plateaus on continents must be very largely balanced and neutralized by corresponding deficiencies of density in the crust beneath, which in turn explains how the larger relief is sustained. This is the theorem of isostasy. Various hypotheses in regard to the magnitude and distribution of these deficiencies in density under the continents, of excesses under the oceans, may be made, and the deflections recomputed on these successive suppositions and compared with the observed deflections.

[1] At the recent meeting of the Geological Society of America, December 30, 1913, to January 1, 1914, Professor W. H. Hobbs gave a paper on "A Criticism of the Hayfordian Conception of Isostasy Regarded from the Standpoint of Geology." The writer did not have the pleasure of hearing this paper, but it is clear that Professor Hobbs has attacked independently the same problems as here discussed.

The difference is the residual error due to the partial incorrectness of a hypothesis. The exactly correct hypothesis would reduce all residual errors to zero except for the errors of observation and computation. A hypothesis which approximates to the truth will give small residual errors. In a large mass of data the sum of the squares of the residuals as derived from different hypotheses serves as a test of the relative agreement of the hypotheses with nature, that hypothesis applying best for which the sum of the squares is a minimum. In all of the complete solutions a uniform distribution of compensation was assumed to exist from the surface to the bottom of the zone of isostatic compensation. That is, if the column under a certain portion of land was 3 per cent lighter than under a certain portion of water, then it was assumed that at any and every depth the two columns differed in density by 3 per cent. The differences abruptly terminate at the level where the two columns, the long but light land column and the short but heavy sea column, become of equal weight. At the level of this surface isostatic compensation is complete and there is hydrostatic equilibrium.

A tabulation of the probabilities of these hypotheses as applied to the whole of the United States is as follows:

TABLE III

Hypothesis	Sum of Squares of 765 Residuals
Solution B (extreme rigidity; depth of compensation infinite)	107,385
Solution E (depth of compensation 162.2 km.)	10,297
Solution H (depth of compensation 120.9 km.)	10,063
Solution G (depth of compensation 113.7 km.)	10,077
Solution A (depth of compensation zero)	18,889

The first investigation, that of 1906, favored Solution G, the final, that of 1909, as shown in this table, favored H. The most probable depth on the hypothesis of uniform compensation with depth and of equal depth of compensation for the whole United States was a little greater, being 122.2 km., 76 miles. It is seen, however, that there is but little change in the sum of the squares for a considerable range in the assumed depth. Further, Hayford states that the hypothesis of all compensation being attained in a 10-mile stratum whose bottom is at a depth of 35 miles is about as probable as the solution which he adopted.[1] Other variations in the hypothesis are also possible with about the same probable error.[2]

[1] 1906, p. 151. [2] 1906, p. 153.

A distribution suggested by Chamberlin, of compensation greatest a little below the surface and diminishing to nothing at 178.6 miles, is also about as probable. Hayford therefore does not claim that his geodetic studies determine with precision the nature or depth of the distribution of compensation. The figure of 76 miles should therefore be used always with this reservation.

The residuals were classified into fourteen geographic groups. The most probable depths of compensation indicated for the several groups range from 66 to 305 km. According to Hayford, the evidence from these groups is, however, so weak and conflicting that he sees no indication that the depth of compensation is not constant over the whole area investigated.[1] He notes that, so far as the evidence goes, it indicates the depth of compensation to be greater in the eastern and central portions of the United States than in the western portion.[2] The subject is one which will be taken up later in the discussion of geodetic results.

In regard to the completeness of compensation, Hayford states:

From the evidence it is safe to conclude that the isostatic compensation is so nearly complete on an average that the deflections of the vertical are thereby reduced to less than one-tenth of the mean values which they would have if no isostatic compensation existed. One may properly characterize the isostatic compensation as departing on an average less than one-tenth from completeness or perfection. The average elevation of the United States above mean sea-level being about 2,500 feet, this average departure of less than one-tenth part from complete compensation corresponds to excesses or deficiencies of mass represented by a stratum only 250 feet (76 meters) thick on an average.[3]

It is not intended to assert that every minute topographic feature, such, for example, as a hill covering a single square mile, is separately compensated. It is believed that the larger topographic features are compensated. It is an interesting and important problem for future study to determine the maximum size, in the horizontal sense, which a topographic feature may have and still not have beneath it an approximation to complete isostatic compensation. It is certain, from the results of this investigation, that the continent as a whole is closely compensated, and that areas as large as states are also compensated. It is the writer's belief that each area as large as one degree square is generally largely compensated. The writer predicts that future investigations will show that the maximum horizontal extent which a topographic feature may have and still escape compensation is between 1 square mile and 1 square degree. This prediction is based, in part, upon a consideration of the mechanics of the problem.[4]

[1] 1909, pp. 55–59.

[2] 1906, pp. 143, 146.

[3] 1909, p. 59.

[4] 1906, p. 169.

These conclusions imply a weakness of the crust surprising to the geologist and stand in marked contrast to those figures derived from the study of the deltas of the Nile and Niger. This subject also will be discussed later, as here it is desired to give only a summary statement of the methods and conclusions.

Hayford and Bowie on variations of gravity.—Regarding the relations of variations in gravity to isostasy, Hayford and Bowie state:

> As soon as it was evident that the proper recognition of isostasy in connection with computations of the figure and size of the earth from observed deflections of the vertical would produce a great increase in accuracy, it appeared to be very probable that a similar recognition of isostasy in connection with computations of the shape of the earth from observations of the intensity of gravity would produce a similar increase of accuracy. Logically the next step to be taken was therefore to introduce such a definite recognition of isostasy into gravity computations. Moreover, it appeared that if this step were taken it would furnish a proof of the existence of isostasy independent of the proof furnished by observed deflections of the vertical, and would therefore be of great value in supplementing the deflection investigations and in testing the conclusions drawn from them. In other words, the effects of isostasy upon the direction of gravity at various stations on the earth's surface having been studied, it then appeared to be almost equally important to investigate the effects of isostasy upon the intensity of gravity.[1]

In order to make the computations, the isostatic compensation was assumed to be complete under every topographic feature and uniformly distributed to a depth of 114 km. below sea-level, producing hydrostatic equilibrium at this depth. The mean density of 2.67 was taken as applying to the whole zone to this depth. Under land 3 km. high this gives a density of 2.60 from sea-level to a depth of 114 km.; under ocean 5 km. deep a density of 2.74 from ocean bottom to 114 km. below the bottom.[2]

The authors show that the topography and its compensation for the whole earth must be taken into consideration. On these assumptions the theoretic value of gravity was computed for every station, 124 in the final publication. This computed value is subtracted from the observed value and gives the "new-method" anomaly for each station. The results are shown in Fig. 5.

[1] Hayford and Bowie, 1912, p. 5.

[2] Hayford and Bowie, 1912, pp. 9, 10.

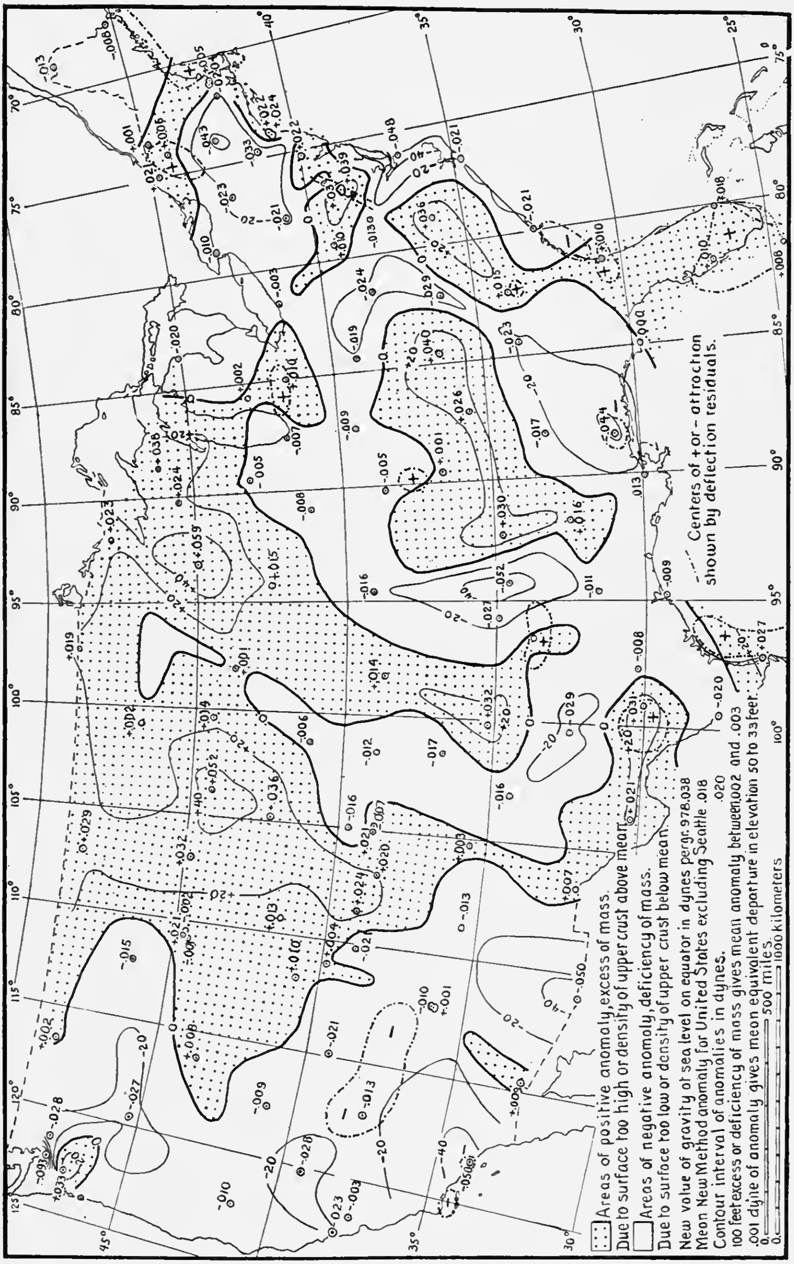

FIG. 5.—Lines of equal anomaly for new method of reduction, after Bowie

Of the two other principal methods of gravity reduction which have been previously used, the Bouguer reduction takes no account of isostatic compensation, postulating a high rigidity of the earth's crust, and neglects all curvature of the sea-level surface. The "free-air" reduction assumes that each piece of topography is compensated for at zero depth. These two reductions correspond thus to the limiting solutions tried for deflections of the vertical. The sum of the squares of the new method anomalies, when compared respectively with the similar sums derived from the hypothesis of rigidity and the hypothesis of compensation at depth zero, shows that the assumption of isostatic compensation uniformly distributed to a depth of 114 km. gives on the average smaller anomalies; is therefore much more probable and yields a more accurate value for the intensity of gravity. The mean anomaly of all stations in the United States without regard to sign, omitting the exceptionally large anomalies of the Seattle stations, is as follows:

New method	0.018 dyne[1]
Bouguer	0.063
Free air	0.028

The value of gravity for the United States Coast and Geodetic Survey office at Washington was determined as 980.112 dynes per gram. The mean new-method anomaly is consequently about 0.00002 of the value of gravity. The probable error of observation and computation is about 0.003 dyne. The errors may, however, frequently exceed 0.004 dyne and in rare cases may be as great as 0.010 dyne.[2] The fact that these measures of gravity are the forces acting on one gram will be understood through the rest of the paper.

Of the 124 stations, 32 have anomalies between 0.020 and 0.030, 12 have anomalies between 0.030 and 0.040.[3] Still smaller numbers of stations have higher anomalies. These anomalies measure departures in the earth's crust from the conditions of isostasy which were postulated. In the interpretation of the anomalies in terms of mass it is shown that a small excess of mass immediately below

[1] Bowie, 1912, p. 12.

[2] Hayford and Bowie, 1912, p. 79; Bowie, 1912, p. 13.

[3] Bowie, 1912, p. 13.

the station or a large excess at great depth or to one side may have the same effect. Therefore it is necessary to speak of the net effective excess or deficiency of mass.[1] A table is given showing these relations, and as a mean working hypothesis it is assumed that ordinarily each 0.0030 dyne of anomaly is due to an excess or deficiency of mass equivalent to a stratum 100 ft. thick. In the final paper it is concluded:

From the evidence given by deflections of the vertical the conclusion has been drawn that in the United States the average departure from complete compensation corresponds to excesses or deficiencies of mass represented by a stratum only 250 feet thick on an average. The gravity determinations indicate this average to be 630 feet instead of 250 feet. In neither case is the average value determined or defined with a high grade of accuracy. The difference between the two determinations of the average value is therefore of little importance. The determination given by the gravity observations is probably the more reliable of the two. Each determination is significant mainly as showing that the isostatic compensation is nearly perfect.

The average elevation in the United States above mean sea level is about 2,500 feet. Therefore, from gravity observations alone the compensation may be considered to be about 75 per cent complete on an average for stations in the United States.[2]

This conclusion implies a somewhat greater rigidity to the crust than that which is stated for the deflections of the vertical, but in regard to the maximum horizontal extent which a topographic feature may have and still escape compensation the authors still express the belief that the limit is between one square mile and one square degree. "It appears from the inconclusive evidence furnished by the gravity observations that the radius of this area is probably less than 18.8 kilometers."[3]

This review of the work of Hayford on deflections of the vertical, and of Hayford and Bowie on the gravity anomalies has been given in order that the methods of the work, its bearings on the strength of the crust, and the conclusions which were reached, may be perceived. It is seen that a large difference of view as to the strength of the crust exists between this interpretation from the geodetic evidence and that from the geologic. In the following pages will be

[1] Hayford and Bowie, 1912, pp. 108–12; Bowie, 1912, p. 22.

[2] Bowie, 1912, pp. 22, 23.

[3] Hayford and Bowie, 1912, p. 102.

given a discussion which it is thought brings out certain errors in the conclusions drawn from the geodetic work and thereby reconciles the two lines of evidence.

REGIONAL VERSUS LOCAL DISTRIBUTION OF COMPENSATION

Conclusions on this topic by Hayford and Bowie.—Under this heading Hayford and Bowie state:

The question whether each topographic feature is completely compensated for by a defect or excess of mass exactly equal in amount directly under it, or whether the topographic feature is compensated for by a defect or excess of mass distributed through a more extensive portion of the earth's crust than that which lies directly beneath it, is a very important one. The theory of local compensation postulates that the defect or excess of mass under any topographic feature is uniformly distributed in a column extending from the topographic feature to a depth of 113.7 kilometers below sea level. The theory of regional compensation postulates, on the other hand, that the individual topographic features are not compensated for locally, but that compensation does exist for regions of considerable area considered as a whole.

In order to have local compensation there must be a lower effective rigidity in the earth's crust than under the theory of regional compensation only. In the latter case there must be sufficient rigidity in the earth's crust to support individual features, such as Pikes Peak, for instance, but not rigidity enough to support the topography covering large areas.

Certain computations have been made to ascertain which is more nearly correct, the assumption of local compensation or the assumption of regional compensation only. In making such computations it is necessary to adopt limits for the areas within which compensation is to be considered complete. A reconnoissance showed that the distant topography and compensation need not be considered, for their effect would be practically the same for both kinds of distribution. As a result of this reconnoissance it was decided to make the test for three areas, the first extending from the station to the outer limit of zone K (18.8 kilometers), the second from the station to the outer limit of zone M (58.8 kilometers), and the third, to the outer limit of zone O (166.7 kilometers).[1]

The average anomaly with regard to sign by the new method with local compensation, and the average anomaly by each of the three new-method reductions with regional distribution of the compensation are respectively −0.002, −0.001, −0.001, and −0.002 dyne. The means without regard to sign for the different distributions of the compensation are respectively, 0.020, 0.019, 0.019, and 0.020 dyne. These mean anomalies give only negative evidence.[2]

[1] Hayford and Bowie, 1912, p. 98. [2] Bowie. 1912, p. 22.

The problem may be tested in another way.

If local compensation be true, an unusually high mountain is underlain by unusually light matter and the intensity of gravity at a station on its top is less than if the mountain was supported by regional compensation and had matter of the mean regional density below it.

If the station is much below the average level of a mountainous region, local compensation implies, on the contrary, denser matter beneath and a higher value of gravity than would be given by regional compensation. These relations result in the following principle: For stations above the mean level, if local compensation be nearer the truth the hypothesis of regional compensation would tend to show its error by large negative anomalies. If regional compensation be nearer the truth, the hypothesis of local compensation would tend to show its error by giving large positive anomalies. For stations below the mean level the reverse would be true. But for any individual station other departures from the truth of that hypothesis of isostasy which gives the basis for the calculations may have greater influences and give larger anomalies than the question to be tested. Following this principle it is stated:

There are 22 stations in the United States in mountainous regions and below the general level and the means, with regard to sign, of the anomalies by the four methods of distribution are 0.000, +0.001, +0.003, and +0.005 dyne, while the means without regard to signs are respectively 0.017, 0.017, 0.018, and 0.019 dyne. For the 18 stations in the United States in mountainous regions and above the general level the means, with regard to sign, of the anomalies by the several methods of distribution of the compensation are +0.003, +0.003, 0.000, and −0.10 dyne. The means, without regard to sign, are respectively 0.018, 0.018, 0.017, and 0.020 dyne.

The mean, with regard to sign, of the anomalies for the stations at each of the two mountain groups, indicates that the theory of regional distribution of compensation to the outer limit of zone O, 166.7 kilometers is far from the truth. So far as may be judged from the other average anomalies no one method seems to have any decided advantage (see pp. 98–102 of *Special Publication No. 10*).[1]

Review and analysis of the evidence.—The present writer does not see in these computations any support for the hypothesis of local

[1] Bowie, 1912, p. 22.

compensation of the topography to between limits of one square mile and one square degree with the added suggestion of a radius less than 18.8 km., which has been advanced on other pages by the authors.[1] These figures merely show that, to the outer limit of zone M, radius 58.8 km., and probably to outer limit of zone N, radius 99 km., one method is as good as another for purposes of computation, which is not true in nature. The errors introduced by observation and computation, the errors introduced by the lack of recognition necessary in the preliminary hypothesis regarding the irregularities in the depth and distribution of compensation—these produce effects which overshadow the small systematic differences due to the hypotheses of local versus regional compensation. For the outer limit of zone O, radius of 166.7 km., a real distinction does, however, begin to appear in the data for the two groups of mountain stations. It is, however, very small and based upon a rather too limited number of stations to give quantitative reliability to the mean. Furthermore, as discussed in detail under a later heading, there is quite possibly a real difference between the limits of regional compensation and depth of compensation in the mountain regions of the West compared to other parts of the continent. Evidence drawn from the Cordillera cannot, therefore, be applied safely to the other portions of the United States.

Let the assumption be introduced that the limits of regional compensation are variable, ranging from 100 to 500 km. in radius. Such variable limits may well exist because of several factors; first, because of a real variability in the strength of the crust; second, because the greater vertical stresses could be carried only by smaller areas. In regions of mountainous relief due to folding, or of high anomalies due to great irregularities of density, the mean size of unit areas should therefore be less. On the whole the anomalies as well as the relief appear to be somewhat greater over the western United States. Third, in regions of recent block faulting or warping the stresses have presumably been lessened from what they were immediately before the movement. Such diminution of strain could take place by the breaking-up of a large unit area of crust into smaller units with differential movement among them, as

[1] Hayford and Bowie, 1912, p. 102.

well as by vertical movement of the whole area to a level best satisfying the stress. The western United States is known to be such a region, which in the late Tertiary and up to the present has been markedly affected by block faulting and differential vertical movements.

Suppose, then, that the mean radius of regional compensation in a mountainous region is 300 km. but that unit areas exist ranging in radius from 100 to 500 km. Of mountain stations located at random, a fraction of the total number would be situated within or near areas where regional compensation did not extend to 166.7 km. Let the stations be divided into one group consisting of those below the mean regional elevation and another group above the mean regional elevation. Let the anomalies be computed successively according to hypotheses of regional compensation to successive limits and the mean of the group for each limit be taken. This is the test applied by Hayford and Bowie. It has been seen that for radii of 18.8 and 58.8 km. the results are indeterminate. For a larger radius the group anomaly might be expected to show an increase as soon as the assumed radius exceeded the actual radii of a part of the areas. Consequently, if the hypothesis be true that the areas of regional compensation are variable in size, the mean anomalies of the two groups of 22 and 18 stations, found with regard to sign to be +0.005 and −0.010 respectively for radius of 166.7 km., do not show that regional compensation on the whole does not exist to those limits. It may indicate only that some areas are less than that radius. The mean radius of regional compensation may be 166.7 km. or possibly even larger. Other tests must therefore be sought which will give a more conclusive answer.

Further, it is to be noted that the mean anomalies with regard to sign for the hypothesis of regional compensation to radius of 166.7 km., although somewhat greater than for the other hypotheses, are yet of the same order of magnitude; and in all cases are but a fraction of the mean anomaly without regard to sign. Apparently, then, the assumption of regional compensation to 166.7 km. introduces a smaller error than the assumption of uniform and complete compensation with an average specific gravity of 2.67 to a constant depth of 114 km.

The test by adjacent stations at different elevations.—There is, however, another way of using the data given for stations situated well above and below the mean elevation of mountainous regions. If a pair of stations be taken close together, one far above the mean elevation, the other far below, they will presumably, because of their juxtaposition, be affected in much the same way by the errors incident to the hypothesis of uniform compensation through a depth of 114 km., with complete compensation at that depth. In order that good results may be obtained, however, the specific gravity of the local rocks should be carefully determined in order to have a correction for the mass between the stations. The parts of the anomalies due to the irregularities and incompleteness of compensation will ordinarily have the same sign and be of nearly the same value at the adjacent stations. This is indicated by the contour lines of Fig. 5, which show that in the same region the anomalies are of sufficiently regular gradation in magnitude to make the drawing of contour lines possible. The parts of the anomalies at the high and low stations due to errors in the hypothesis of local or regional compensation will, however, be of opposite sign. If, then, the algebraic difference of the anomalies for such a pair of stations be computed for successive hypotheses of broader regional compensation, the part of the anomalies due to *vertical* imperfection of the hypothesis will be largely eliminated. The algebraic difference measures the *horizontal* imperfection of the hypothesis. That hypothesis is favored whose assumed radius of regional compensation gives a minimum value to this algebraic difference. This test may be made by combining data given on p. 100, Hayford and Bowie, with p. 15, Bowie; although, because of incompleteness of the tables, this combination gives the data for only a few of the properly situated mountain stations. The best couple of stations for the application of this test consist of 42, Colorado Springs, and 43 Pikes Peak. Somewhat more distant stations—44, Denver, and 45, Gunnison—may also be added to the group. The tabulation is shown on p. 161 (Table IV).

It is seen that for three of the four Colorado stations the absolute value of the anomaly is least with regional compensation to 166.7 km. For the fourth station it remains practically constant for

all the cases. The anomalies were not computed for greater radii. The more convincing argument, however, for regional compensation to at least 166.7 km. radius in the vicinity of Pikes Peak is the fact that the *algebraic difference* of the anomalies between the top and bottom of the mountain, stations 43 and 42, is less than one-half for regional compensation to 166.7 km. radius than for the corresponding value given by the hypothesis of local compensation. The decrease in the difference is furthermore progressive with each

TABLE IV

Number and Name of Station	Elevation of Station in Meters	Distance from Mean Elevation in Meters within 100 Miles	Anomaly with Regional Compensation within Outer Limit of			
			Local Compensation. Radius 0.0 Km.	Zone K, Radius 18.8 Km.	Zone M, Radius 58.8 Km.	Zone O, Radius 166.7 Km.
COLORADO						
42. Colorado Springs......	1,841	−420	−0.009	−0.009	−0.010	−0.010
43. Pikes Peak.....	4,293	+2,035	+ .019	+ .011	+ .006	+ .002
44. Denver........	1,638	−574	− .018	− .016	− .009	− .001
45. Gunnison......	2,340	−380	+ .018	+ .021	+ .026	+ .016
Mean of 42, 44, and 45.........	....	−458	− .009	− .004	+ .007	+ .005
Algebraic Difference 43–42.......	....	....	+ .028	+ .020	+ .016	+ .012
43−(mean of 42, 44, 45)........	....	....	+ .028	+ .015	− .001	− .003
ARIZONA						
68. Yavapai.......	2,179	+512	− .001	− .001	− .001	− .009
69. Grand Canyon..	849	−824	− .012	− .011	− .011	− .021
Algebraic difference 68−69......	....	....	+0.011	+0.010	+0.010	+0.012

assumed widening of the zone. The result of adding the more distant stations, 44 and 45, favors regional compensation more markedly but is indeterminate between M and O. It would seem, then, that the front range of the Rocky Mountains in Colorado is upheld above the surrounding plains and parks by virtue of the rigidity of the earth.

The two stations in Arizona at 68 and 69 are well situated also to test the question of local versus regional compensation, but the

difference in the anomalies in this case is so nearly constant as to give an indeterminate answer. In the absence of more detailed statements by Hayford and Bowie the reason why the anomaly at the Grand Canyon station 69 reaches a larger *negative* value for regional compensation to 166.7 km. than for more limited compensation is not evident. The usual rule is that the progressive change in the anomaly for stations below the regional level for successive assumptions of wider regional compensation is by increments with a *plus* sign. Here, on the contrary, the change in the limits from zone M to zone O involves a *minus* increment of 0.010 in the anomaly. The cause of this reversal of sign, which the writer does not understand, seems in this case to be the cause of the indeterminate result.

Another line of evidence as to the effective limits over which the rigidity of the earth may extend is derived from a study of the grouping of the deflections of the vertical shown in illustrations 2, 3, 5, 6, Hayford, 1909, and the lines of equal anomaly for the new method of reduction, illustration No. 2, Bowie, 1912, the latter giving the basis for Fig. 5 of this article.

The test by areas of grouped residuals.—Illustration No. 5, Hayford, 1909, shows the grouping of the residuals of solution H for the north and south components of the deflections. An area with a plus sign corresponds to an excess of density to the south, or deficiency to the north. An area with a minus sign corresponds to a deficiency of density to the south, or excess to the north. A north-south chain of stations is therefore best for ascertaining the limits of the areas of north-south deflection of like sign. Such a belt extends across the United States between long. 97° and 98°, showing 9 areas covering 1,620 miles. The mean intercept is therefore 180 miles. This mean intercept must be somewhat less than the mean diameter.

Illustration No. 6, Hayford, 1909, shows the grouping of the residuals of solution H for the east and west components of the deflections. An area with a plus sign corresponds to an excess of density to the east, or deficiency to the west. An area with a minus sign corresponds to a deficiency of density to the east, or excess to the west. An east-west chain of stations is therefore best

for ascertaining the limits of the areas of like sign. Such a belt extends across the United States between lat. 38° and 39°.

The following adjustments in groups seem, however, fair to make, considering the lack of exact accuracy in any one station. At Cincinnati is a station showing small residuals opposite in sign to the stations on each side. If this is overlooked, three small groups become one of average size. In central Kansas a small minus area depending on a single observation may be likewise omitted. In western Colorado several small areas depending each upon two observations had their number diminished by one. The same was done in California. This gave 14 areas extending over 2,580 miles, a mean individual intercept of 184 miles. If 16 areas be taken, a mean value is derived of 161 miles. More weight, it is thought, is to be attached to the determination of 184 miles, and this is supported by the 180 miles shown by the north-south chain of stations.

The areas of like sign are between centers of excess and defect of mass. They are not, therefore, coincident with the areas of excess and defect, but in discussing the average size of areas, the one may be used as a measure of the other.

It may be concluded, therefore, that the deflections of the vertical show areas with departures from isostatic equilibrium in one direction and these areas average about 180 miles, 290 km., in mean intercept. The mean diameters of the areas of like sign are presumably somewhat greater. This would make the mean radius of areas of regional compensation, as indicated by similarity of sign among residuals, at least 166.7 km.—the radius of the outer limit of zone O used in the discussion of the gravity anomalies.

If we turn now to the anomalies shown by the determinations of gravity, Fig. 5, adapted from Bowie, shows their segregation into areas of like sign. The mean value without regard to sign for all stations excluding Seattle is 0.018 dyne per gram. Including the two Seattle stations the mean is 0.020 dyne. Between the contours for −0.020 and +0.020 lie tracts where the anomalies are within the mean limits. The areas of exceptionally large anomalies are above those limits. It is only these which form on this illustration well-defined inclosed areas, but even these are far from

regular in outline. The areas showing positive anomalies of more than 0.020 dyne were estimated roughly to average 130 by 240 miles across, a mean diameter of 175 miles. The areas showing negative anomalies of more than 0.020 dyne were found to average roughly about 190 by 250 miles, a mean diameter of 220 miles. The long narrow connections were neglected in making this estimate. Unit areas of more than mean anomaly may therefore be taken to average about 200 miles or 320 km. in diameter. The mean radius is therefore approximately that of the outer limits of zone O, 166.7 km.

The figures, although they correspond fairly closely to those derived from the deflections of the vertical, cannot in reality be very well compared, since these are areas selected because the anomaly rises above a certain magnitude; the others represent, on the contrary, a succession of contiguous areas between centers of excess and defect in mass without reference to magnitude. Apparently some influence blurs out the limitations of areas of small gravity anomaly. This will be discussed in a later part.

Now assume for the moment that isostatic compensation is uniform to the bottom of the zone, as postulated by the hypothesis; that is, that the residuals and anomalies are due to excesses or defects of mass which are uniformly distributed. Then, over any one area of excess or deficiency of mass, the deflections around it and anomalies within it signify a departure from compensation in one direction. This is a regional departure. If the strength of the crust was so small that it was able to support notable departures from compensation over areas of only one square degree or less, then these large unit areas could not exist. A vertical warping up or down would immediately take place until the broad region as a whole lay so close to complete compensation that its surface irregularities became subdivided into subordinate positive and negative areas of the limiting size. The sum of the excesses and defects of mass would approach zero in broad areas containing many unit departures. It would seem, therefore, that the geodetic results shown in Fig. 5, instead of indicating local compensation to limits of less than one square degree, show on the contrary a ready

capacity of the crust under the United States to carry over areas of from 5 to 10 or 15 square degrees, and exceptionally over even larger areas, departures from equilibrium greater than the mean. This agrees in order of areal magnitude with the Nile and Niger deltas. However, the influence of irregularity in the distribution of compensation with depth, and the magnitude of stress per unit area remain to be investigated.

[*To be continued*]

THE STRENGTH OF THE EARTH'S CRUST

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PART III. INFLUENCE OF VARIABLE RATE OF ISOSTATIC COMPENSATION

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INTRODUCTION AND SUMMARY

The work of Hayford on the deflections of the vertical, and of Hayford and Bowie on the anomalies of gravity, has supplied the geodetic data from which future work must start. As an initial basis to guide their work, it was desirable to assume the hypothesis that isostatic compensation was complete for each topographic irregularity, giving local compensation, and that it was uniformly distributed to a constant depth. The actual results may then be compared to this ideal of local, uniform, and complete isostasy and the degree of departures noted, as given by residuals and anomalies.

In Part II the subject of the regional distribution of compensation was examined and the conclusion was reached that the crust was sufficiently rigid to bear such mountains as Pikes Peak without requiring special compensation below. In general it is thought compensation in mountain regions extends to more than 200 km. and in some regions to more than 400 km. In this part are considered the effects of variations in the vertical distribution of

compensation and the degree to which such variability may give rise to anomalies and residuals without signifying incompleteness of compensation in the column as a whole or regional departures from isostasy.

In order to show the limits of variation in density which are to be expected, the specific gravity of rocks is first considered. Figures are computed for the mean specific gravity of igneous rocks and the three types of sediments. It is shown that the range of variation is an important factor. Under the subject of the relations between mass and the distance of mass upon anomalies, the effects are computed of unit masses at various depths and extending various distances.[1] This lays the basis for considering the influence of the specific gravity of the surface geologic formations upon the difference between the mean anomalies for stations on pre-Cambrian and those on Cenozoic areas. It is found that the greater density of the older rocks accounts for a part and another part is accounted for by their resistance to erosion. This still leaves, however, large outstanding regional variations not related to surface geology or topography and requiring some other explanation. To that end criteria are discussed for the recognition and separation of the effects of mere variable vertical distribution of compensation on the one hand, from partial regional absence of isostasy on the other. It is concluded from the application of these criteria that the anomalies are in large part caused by real regional departures from isostasy extending over broad areas. The results are thus

[1] A paper by Gilbert has recently appeared entitled "Interpretation of Anomalies of Gravity" (*Part C, Professional Paper 85, U.S. Geological Survey*, 1913). This did not reach the present writer until after Parts III and IV of this article were in galley proof, so that his results cannot be as fully interwoven into the discussion as would otherwise have been the case. On pp. 30, 31, Gilbert considers the interpretation of anomalies on the assumption of vertical heterogeneity of the crust and shows clearly that moderate variations of density in a vertical direction could explain them. From this he infers that the anomalies may be due in part to such irregularities. This is the topic which is treated in Part III of the present article under the title "Interpretation of Anomalies in Terms of Mass and Depth." The method of reasoning is somewhat different, but although the conclusion reached is the same, the calculations given here are intended to bring out in addition the limitations of area and mass within which that principle applies. It is concluded as a result of the following examination of the evidence that although vertical variations of density are a real cause they are not the major cause of anomalies.

confirmatory of those reached in Parts I and II. In addition, however, it appears that there is a regional departure from isostasy of two orders of magnitude. Loads under the mean value, giving anomalies below 0.018 to 0.020 dyne and estimated to be equivalent to about 750 feet of rock, can be carried over regions of irregular boundaries ranging up to from 1,000 to 2,000 km. across. Over such a broad region the anomalies are of one sign except for some smaller well-defined sub-areas of high anomaly within them which may or may not have the same sign. These smaller areas give a higher order of stress magnitude and are of more restricted dimensions, being measured in hundreds of kilometers. They range in magnitude of anomaly to several times the value of the mean and the equivalent radii of their areas probably average 100 to 200 km. The deflection residuals show by the limits of the areas of like sign that the regional variations of gravity anomalies of this areal magnitude extend over the whole country, but where the amounts of the local anomalies are less in value than the mean they are largely masked on the contour map of gravity anomalies (Fig. 5), because of their superposition upon the broader areas. Presumably a multiplication of the gravity stations would bring them to light as undulations in the contours which show the regional departures.

A final conclusion on the subject of the variable vertical distribution of mass must, however, be deferred until consideration has been given to a hypothesis advanced by Gilbert in his recent paper, that heterogeneities of mass below the zone of compensation may be the cause in major or minor part of the apparent departures from isostasy. This is a subject too large to be considered in this third part of the present article, but it is planned to investigate it in Part V by a method of graphic analysis devised for determining the depth of excesses or deficiencies of mass.

THE SPECIFIC GRAVITY OF ROCKS

For a knowledge of the variations of density likely to occur in rocks it is important to know the range in specific gravities shown by the common rock types. The following figures, except those for shale, are taken from Pirsson's *Rocks and Rock Minerals:*

TABLE V

Rock	Specific Gravity
Granite	2.63–2.75
Syenite	2.6 –2.8
Diorite	2.8 –3.1
Dolerite	3.0 –3.3
Limestone	2.6 –2.8
Sandstone	2.5 –2.7
Shale	2.4 –2.8
Slate	About 2.8

[The specific gravity of shale, although the most abundant of sedimentary rocks, is not given in any of the manuals of geology, but Professor Hobbs, who has read much of this manuscript and to whom the writer is indebted for a number of suggestions, has called attention to the above figure as given by Trautwine. In general, Trautwine and Kent give a somewhat greater range in specific gravities and they average a little lower than those here given. The figures from Pirsson, however, probably express more closely the relation of the petrologic type and the more compact states of rocks to their density. They are, therefore, thought to be better representative of the lithosphere.]

These figures show that notable departures may occur from the mean density of the outer crust and suggest furthermore that 2.67, the mean density used by Hayford, is lower than the actual mean. A more thorough analysis of the subject is therefore needed.

The abyssal igneous rocks and metamorphic rocks are almost without pore space. The sedimentary rocks, on the other hand, possess abundant pore space in their unconsolidated states, very little in their compact states. The latter is the usual mode of occurrence in the older geological formations. The density is therefore a function of both mineral composition and porosity. The chemical compositions of the several rock types and also of the average sediment and the average igneous rock are well known. The mineral compositions are less well known but may be computed with a fair degree of accuracy; the densities, on the contrary, are least commonly reported and the mean densities of the rock types cannot in consequence be closely determined by averaging numerous determinations, as is done for the chemical compositions. It seems desirable, therefore, to compute the densities of the rock types from the chemical and mineral compositions, combining this with the densities of the individual minerals, making a separate correc-

tion for the porosity factor. The data, assembled from various sources[1] and subjected to computation, give the following results:

TABLE VI

COMPOSITION OF AVERAGE IGNEOUS ROCK

Mineral	Percentage
Quartz	12.0
Feldspars	
Orthoclase molecule	22.0
Albite molecule	29.5
Anorthite molecule	8.0
Hornblende and pyroxene	16.8
Mica	3.8
Accessory minerals	7.9
	100.0

TABLE VII

COMPOSITION OF AVERAGE SEDIMENTS

Mineral	Shale	Sandstone	Limestone
Quartz	22.3*	66.8*	2.0
Feldspars			
Orthoclase	18.0	7.0	0.3
Labradorite	12.0	4.5	0.1
Clay	25.0†	6.6†	2.0‡
Limonite	5.6	1.8	0.6
Calcite	5.7	11.1	55.0
Dolomite			35.0
Other minerals	11.4	2.2	5.0
	100.0	100.0	100.0

* The total percentage of free silica.

† Probably sericite in part; in that case the feldspar figure becomes lower.

‡ Two per cent clay takes 0.79 of Al_2O_3. This requires that most of alkalies form non-aluminous hydrous silicates or that 0.81 Al_2O_3 as given by Clarke is too low.

It is thought that the densities without porosity are figures of some value for geodetic computations. The chief error in making the final estimates is in connection with the lack of accurate knowledge regarding the pore space of those sedimentary rocks not used

[1] For data on the mean chemical and mineral composition of rocks see F. W. Clarke, "Data of Geochemistry," *Bull. 491, U.S. Geol. Surv.*, 1911, pp. 30, 31. For specific gravities of minerals see Pirsson, *Rocks and Rock Minerals*, 1908, p. 31; also Dana, *Mineralogy*. For a discussion of pore space see Fuller, "Total Amount of Free Water in the Earth's Crust," *Water Supply Paper No. 160, U.S. Geol. Surv.*, 1906, pp. 59–72.

as building stones, but this affects appreciably the density of only a superficial layer and chiefly of the youngest deposits.

The ratio of shale, sandstone, and limestone in the average sediment in percentage is, according to Mead,[1] shale 80, sandstone 11, limestone 9. The ratio of average porosities in percentage is, according to Fuller,[2] crystalline rocks 0.2, shales 4, sandstones 15, limestones 5. The figure given by Fuller for shale rests upon a single determination of 7.8 per cent by Delesse, and is averaged in by Fuller with slate. Eight per cent porosity will here be assumed as probably a better estimate. This gives the porosity of the average sedimentary rock as 8.5 per cent. The pore space may be taken, following Fuller's estimate, as half filled with water.

From these data the specific gravities are computed to be as follows:

TABLE VIII

SPECIFIC GRAVITIES COMPUTED FROM MINERAL COMPOSITIONS

Rock	No Pore Space Allowed	Pore Space Half Filled with Water
Average igneous rock..	2.80	2.80*
Shale...............	2.69	2.51
Sandstone...........	2.67	2.35
Limestone...........	2.76	2.64
Average sedimentary rock..............	2.70	2.50

*The same figure as used by Chamberlin and Salisbury, *Geology*, I (1904), 538; also by Pirsson, *Rocks and Rock Minerals;* also by G. H. Darwin as the density of the outer crust.

Where Cenozoic deposits occur in thickness, they are considerably compacted except at the surface, but still the mean specific gravity, owing to the abnormal pore space and deficiency in limestones, is doubtless less than 2.50; 2.45 may be taken. It is probable, on the other hand, that the Paleozoic rocks on the whole have somewhat less pore space than this average, especially as the porosity figure for sandstone rests mainly upon determinations for brownstone, a rather porous type; 2.55 may then be taken as the average for Mesozoic and Paleozoic formations. The pre-Cambrian

[1] "Redistribution of the Elements in the Formation of Sedimentary Rocks," *Jour. Geol.*, XV (1907), 238–56.

[2] *Loc. cit.*

rocks contain both igneous and sedimentary formations, but the considerable iron ore and metamorphic nature would bring the specific gravity of the sediments somewhat above the average of 2.70 for non-porous sediments. Broad areas of pre-Cambrian probably range therefore between 2.75 and 3.00 in specific gravity. More limited areas, because of a predominance of granite and quartzite, may range as low as 2.70. About 2.67, however, would be a minimum.

As these are merely averages it is better in basing calculations upon them to assume a certain range in density for each figure and to obtain thus a knowledge of the influence of reasonable variations upon the results. The data may then be tabulated as follows:

TABLE IX

ESTIMATED MEAN SPECIFIC GRAVITIES OF GEOLOGIC FORMATIONS

Pre-Cambrian	2.75–2.80
Paleozoic and Mesozoic	2.50–2.60
Cenozoic	2.40–2.50

The range in these specific gravities shows the necessity of considering them in all refined calculations on the anomalies of gravity. In place, however, of using a mean density figure for all stations on formations of a certain geologic age, it would be of much more value to have measurements of the actual surface densities occurring in each area; also estimates by geologists, based on geologic structure and these surface measurements, of the densities extending to the base of the sedimentary rocks of each locality.

It seems probable from the mean density of 2.80 obtained for igneous rocks that the density of 2.67 used by geodesists for the mean density of the zone of compensation is too low. If any variation from the average composition takes place with depth within the limits of 76 miles, it is likely to be a variation toward more basic and heavier rocks. Assuming, however, an average uniformity of chemical composition, the opposing effects of temperature and pressure remain to be considered. Using the coefficient of expansion of the average igneous rock computed by W. H. Emmons,[1] 0.000,019,9 for 1° C., and a temperature gradient of

[1] Chamberlin and Salisbury, *Geology*, I (1904), 547.

1° F. for 60 ft. in depth, gives an aggregate expansion of 3.6 per cent to the outer 76 miles. Using 6,500,000 as the modulus of cubic compressibility of the average rock in pound-inch units[1] gives a total compression of 3.7 per cent to the outer 76 miles due to pressure; that is, the volume effects of heat and pressure practically offset each other within the zone of isostatic compensation. Therefore 2.80 appears to be the lowest mean figure which should be taken. The use of 2.67 as a mean figure requires for isostatic equilibrium a density of but 2.60 extending to a depth of 76 miles under land 3 km. high, a figure lower than the specific gravity of granite.

INTERPRETATION OF ANOMALIES IN TERMS OF MASS AND DEPTH

Suppose that the zone of isostatic compensation is not of uniform density under any one station, but contains masses of variable density irregularly distributed. Let these masses be of considerable thickness and area as compared to the depth of the zone of compensation. Suppose that the topography is so adjusted to the aggregate density that the pressures are everywhere equal at the bottom of the zone of compensation. Abnormally light masses would then have to be balanced by abnormally heavy masses in the same column. There would still be deflections of the vertical and anomalies of gravity because gravitation varies inversely with the square of the distance, the upper and adjacent masses of abnormal density affecting the station more than those more distant ones of opposite abnormality lying vertically below the upper. The residuals from deflection and gravity measurements would under such an arrangement measure strains within the outer crust but not upon its bottom. The strains, if produced by abnormalities in the upper parts of the crust, would further be proportionately smaller and yet give rise to residuals of a certain magnitude than if produced by abnormalities in the lower parts of the crust. This aspect of the problem must be investigated before any final significance regarding the strength of the crust can be attached to the grouping of residuals discussed under the

[1] F. D. Adams and E. G. Coker, *An Investigation into the Elastic Constants of Rocks, More Especially with Reference to Cubic Compressibility*, 1906, p. 67.

last part of Part II. It leads to a consideration of the relations between mass, distance, and anomaly.

Under the title of "Interpretation of Anomalies in Terms of Masses"[1] Hayford and Bowie show that the excesses and deficiencies of mass to a great distance have an effect upon the gravity anomalies and that therefore the guarded expression "net effective excess (or deficiency) of mass" is necessary for correctness. They give the following tabulation to show the influence of uncompensated masses in the crust in giving gravity anomalies when the gravity is computed on the assumption of isostasy:[2]

TABLE X

Each tabular value is the vertical attraction in dynes produced at a station by a mass equivalent to a stratum 100 ft. thick, of density 2.67, and of the horizontal extent indicated in the left-hand argument, if that mass is uniformly distributed from the level of the station down to the depth indicated in the top argument and from the station in all directions horizontally to the distance indicated in the left-hand argument.

Radius of Mass	Depth				
	1,000 Ft.	5,000 Ft.	10,000 Ft.	15,000 Ft.	113.7 Km.
1,280 m. (the outer radius of zone E)	0.0029	0.0018	0.0011	0.0008	0.0000
166.7 km. (the outer radius of zone O)	0.0037	0.0034	0.0034	0.0034	0.0024
1,190 km. (or 10°40′, the outer radius of zone 10)	0.0040	0.0037	0.0037	0.0037	0.0034

On p. 111 it is concluded by these authors that the best working hypothesis is to take

each 0.0030 dyne of anomaly as due to an excess (or deficiency) of mass equivalent to a stratum 100 ft. thick. This working hypothesis is equivalent, as may be seen by inspection of the table just given, either to the assumption that the excess (or deficiency) of mass is uniformly distributed to a depth of 113.7 kilometers and extends to a distance of more than 166.7 kilometers and less than 1,190 kilometers from the station, or that it extends to a distance of 166.7 kilometers from the station and is distributed to an effective mean depth of more than 15,000 feet and less than 113.7 kilometers, or the working hypothesis may be considered to be a combination of these two assumptions.

The mean anomaly of 0.018 dyne, interpreted on this basis of 0.030 dyne being taken as equivalent to 100 ft. of mass, gives a

[1] Hayford and Bowie, p. 108. [2] *Ibid.*, 1912, p. 109.

mean departure from isostatic compensation amounting to 600 ft.; given more exactly by Bowie as 630 ft.

It is seen from the quoted statement that the authors accept, first, as one alternative a very widespread regional net excess (or deficiency) of mass uniformly distributed in depth; or, second, a somewhat broad regional distribution but confined to the outer part of the zone of compensation; or, third, some combination of the two assumptions.

The first assumption would throw a real strain upon the bottom of the zone of compensation and signifies regional compensation to limits very far beyond those stated elsewhere by the authors. It is therefore inconsistent from that standpoint, but gives a smaller vertical load and consequently a smaller vertical departure from the level giving isostatic equilibrium than would a more limited area. If, for example, it be assumed that the radius of the zone limiting regional compensation is 58.8 km., which is about the maximum limit for regional compensation which Hayford allows elsewhere; then it may be computed that for uniform distribution of the excess (or deficiency) of mass to a depth of 114 km., a mass equivalent to 100 ft. of density 2.67 corresponds to an anomaly of but 0.0013 dyne instead of 0.0030. This would, for a mean anomaly of 0.018, signify an average departure over the United States of 1,380 ft. from the level giving isostatic equilibrium, instead of 600 ft.

The second assumption, that the excess (or deficiency) is in the outer part of the crust, gives also a much higher anomaly for a unit mass than would an equally permissible assumption that the excesses or deficiencies occurred at various levels and on the average were at a depth of one-third or one-half of the zone of compensation. The relationship of anomalies to geologic formations, to be discussed later, shows certain variations in density in the outer crust, but the greater parts of the anomalies are not due to this cause. From the previous discussion on the limits of regional compensation it would seem that, on the assumption that the excesses or deficiencies of mass are on the whole uniformly distributed, 0.0024 would be an appropriate figure to use as the mean anomaly for unit thickness of mass. The highest anomalies, however, are

probably better interpreted by 0.0030 as a divisor, since as a class they must be assumed as due to excesses or deficiencies of mass which are both near and large. This does not mean, however, that the larger masses are not assumed as scattered uniformly, according to the laws of chance, through the crust. It is seen, then, that Hayford and Bowie have favored those interpretations which gave a large anomaly per unit mass and have ascribed the total anomaly as on the average to be interpreted on this basis, obtaining thereby a smaller figure as the mean departure in feet from the level for perfect compensation. They have not discussed, furthermore, in the text the influence of deeper-seated variations of density, which might give considerable residuals, nor the possibility that departures from the mean density in opposite directions might balance each other so as to give equal pressures at the bottom of the zone of compensation. The latter case will not seem improbable to the geologist. The great batholiths of the Archean appear to make a universal floor in the crust. They range in composition from granites to gabbros and have come to rest at various levels. Light and heavy masses may well be irregularly distributed in the same vertical cylinder. If at the time of origin the whole were too heavy, a tendency would have arisen for the column to sink until equilibrium was attained. If the whole, on the contrary, were too light, the column would have tended to rise until a heavier base balanced the lighter mass above. Thus, if irregular distribution of density arose as the result of vertical igneous intrusion, the whole region would tend to seek that level where the irregularities would balance.

In order to gain quantitative ideas as to this possibility of partly explaining the anomalies, the writer has made calculations on the following assumptions. A station is situated upon the axis of a vertical cylinder extending from the station to a depth of 114 km. The radius is taken successively at 58.8, 166.7, and 1,190 km. Let such a cylinder be divided into five equal cylinders by horizontal planes. Let each of the five be equivalent in mass to a cylinder of the same radius but only 100 ft. in depth and of density 2.67; in other words, the unit mass as used by Hayford and Bowie. What will be the attraction in dynes per gram pro-

duced at the station by each cylinder respectively?[1] The results are as follows:

TABLE XI

VERTICAL ATTRACTION IN DYNES ON ONE GRAM AT STATION BY CYLINDER 22.8 KM. THICK, DENSITY 0.00357, EQUIVALENT IN MASS TO THICKNESS OF 100 FT. AT DENSITY 2.67

No. of Cylinder	Depth in Km. from Station to Top of Cylinder	Attraction for Radius of 58.8 Km.	Attraction for Radius of 166.7 Km.	Attraction for Radius of 1190 Km.
I.	0.0	0.0031	0.0032	0.0036
II.	22.8	0.0017	0.0028	0.0035
III.	45.6	0.0010	0.0024	0.0035
IV.	68.4	0.0007	0.0020	0.0035
V.	91.2	0.0005	0.0017	0.0034

The results for radius 58.8 km. show that masses of this size situated near the bottom of the zone of compensation exert but a fraction of the influence given by equivalent masses near the surface. A balancing of light and heavy masses in a column of this radius would give isostasy at the base and yet produce notable anomalies. For radius 166.7 km. the importance of depth is much diminished. For radius 1,190 km. it practically disappears. This means that a wide regional variation in depth with plus and minus departures from the uniform density, the light and heavy layers balancing, would not produce anomalies provided, as stated, there was isostatic equilibrium at the base.

To give a somewhat extreme illustration; suppose that the upper cylinder, I, is 2 per cent lighter than the mean density of

[1] The formula for making these computations was kindly worked out for me by Professor H. S. Uhler, checking it as given by B. O. Pierce, *Newtonian Potential Function*, p. 8. It is as follows:

$$F=2\pi\rho\gamma\left[\sqrt{a^2+c^2}-\sqrt{a^2+(c+h)^2}+h\right].$$

in which

F = force in dynes per gram.
ρ = density, in this case = 0.003,57.
γ = constant of gravitation = 0.000,000,066,58.
a = radius of cylinder.
c = distance on axis from station to top of cylinder.
h = depth of cylinder; in this case 22.8 km.

For radii of 58.8 and 166.7 km. no correction need be made for curvature of the earth's surface. For a = 1190 km. an empirical correction was obtained by comparing the results with Hayford's computations.

The writer overlooked until later the fact that Hayford and Bowie also give this formula with a different notation on p. 17 of their work.

2.67 and the lower cylinder, V, is 2 per cent heavier. Let these abnormalities be limited areally to the cylinder. This is a departure in density of 0.054, 15.1 times the density 0.00357. The anomalies will be as follows:

TABLE XII

ANOMALIES DUE TO IRREGULAR VERTICAL DISTRIBUTION OF DENSITY

No. of Cylinder from Table	Density 2 Per Cent from Mean	Anomalies		
		Radius 58.8 Km.	Radius 166.7 Km.	Radius 1190 Km.
I........................	2.616	−0.047	−0.048	−0.054
V........................	2.724	+0.008	+0.026	+0.051
Resultant anomaly........		−0.039	−0.022	−0.003

It is seen from this tabulation that, first, irregular superposed but balanced positive and negative distributions of density up to distances as large as the radii of the areas of grouped residuals could produce at least a considerable part of the anomalies; or, second, actual departures from isostatic equilibrium with the resultant strain on the crust could produce them; or, third, a combination of the two. In the second case, as Hayford and Bowie show,[1] the anomalies could result from a layer a few miles thick adjacent to the station and of very abnormal density; or from deep and regional masses of great volume, but departing only slightly from the mean density. The choice between these several alternatives, or the degree to which they co-operate, must be investigated under the following topics.

RELATIONS OF ANOMALIES TO EXPOSED GEOLOGIC FORMATIONS

The latest data given by Bowie on this subject are shown in Table XIII (p. 222):[2]

These figures of course are not to be regarded as of high precision, as may be seen by comparing the earlier and later results.

[1] *Op. cit.*, Pp. 108–11.

[2] "Some Relations between Gravity Anomalies and the Geologic Formations in the United States," *Am. Jour. Sci.* (4), XXXIII (1912), 237–40.

Hayford and Bowie in their successive publications give the following for the pre-Cambrian and Cenozoic stations, the two groups

TABLE XIII

Geologic Formation	Number of Stations	Mean with Regard to Sign	Mean without Regard to Sign
Pre-Cambrian.......	10	+0.016	0.026
Paleozoic...........	31	−0.003	0.019
Mesozoic...........	20	+0.002	0.015
Cenozoic...........	29	−0.008	0.021
Intrusive and Effusive	11	−0.007	0.015
Unclassified.........	22	+0.011	0.020
All stations.........	123	0.000	0.019

to which the attention will be confined. A few stations of high anomaly must have considerable influence on the result, as most of the stations are used in common in all of the estimates.

TABLE XIV

	Geologic Formation	Number of Stations	Mean with Regard to Sign	Mean without Regard to Sign
Hayford and Bowie, U.S.C. and G.S.	Pre-Cambrian	7	+0.019	0.026
	Cenozoic	20	−0.011	0.021
Bowie, U.S.C. and G.S.....	Pre-Cambrian	9	+0.024	0.024
	Cenozoic	33*	−0.007	0.021
Bowie, *Am. Jour. Sci*.....	Pre-Cambrian	10	+0.016	0.026
	Cenozoic	29	−0.008	0.021

* Fifteen stations have plus anomalies, 17 have minus anomalies.

Bowie's figures in the *American Journal of Science* will be used in the following discussion.

Bowie favors the explanation that these relations of anomalies to geologic formations are due to slight changes of density extending more or less through the zone of compensation and leading to departures from perfect isostasy. The writer, however, is led to favor the view that about one-half of the contrasted anomaly for these two groups is due to a lesser density within the outer mile of crust beneath the Cenozoic stations, as contrasted to the outer mile of crust beneath the pre-Cambrian stations. The remainder of the anomaly it is thought is explained by the ease of erosion of Cenozoic formations, the resistance to erosion of the pre-Cambrian

rocks. The latter consequently tend to stand above the regional levels. They therefore possess surficial excess both of density and volume.

The average thickness of sedimentary rocks if spread uniformly over the globe is thought to be between 2,000 and 2,500 ft.[1] Over the pre-Cambrian areas it must average much less; over the areas of later formations much more. Under the Cenozoic stations assume:

1,000 ft. of sediments at density...........	2.40 to 2.50
4,000 ft. of sediments at density...........	2.50 to 2.60
Giving a total of 5,000 ft. at density........	2.48 to 2.58
With a deficiency of density of.............	0.19 to 0.09

Under the pre-Cambrian stations assume:

5,000 ft. of crystalline rock at density	2.75 to 2.80
An excess of density of...................	0.08 to 0.13

This does not involve the improbable assumption that below the outer 5,000 feet of crystalline rock of density 2.75 to 2.80 the density suddenly decreases to 2.67 and then remains constant throughout the zone of compensation. The vertical density gradient, *if uniform for all points*, has but little effect, it being the *horizontal* variations of density which enter into the problem of isostasy. To maintain conformity with Hayford's figures, therefore, the density 2.67 will be frequently assumed as the mean density of the lithosphere, although the previous discussion shows that it cannot be assumed as the density of the outer mile of crystalline rocks when comparing these to the mile of sedimentary rocks taken as the mean depth underlying the Cenozoic stations.

In comparison with this thickness of 5,000 ft. the average area of formations is very great. A plane sheet of rock 100 ft. thick and of density 2.67, if of indefinite extent, will produce an anomaly of 0.0034 dyne upon a point outside of it, irrespective of the distance to that point. This theory may be applied without gross error to the relation of surface geologic formations to anomalies. If this unit mass be expanded from 100 to 5,000 ft. thickness, the

[1] F. W. Clark, "Data of Geochemistry," *Bull. 491, U.S. Geol. Surv.*, 1911, p. 30.

density will be decreased to 0.053 that of water. The data may then be tabulated as follows:

TABLE XV

Computed Anomalies Due to Densities of Surface Formations

	Deficiencies or Excesses of Density	Anomalies in Dynes per Gram Due to Thickness of 5,000 Ft.
Unit mass	0.053	0.0034
Cenozoic	−0.19	−0.012
	−0.09	−0.006
Pre-Cambrian	+0.08	+0.005
	+0.13	+0.008

These mean anomalies of the pre-Cambrian due to the greater density of the outer 5,000 ft. of rock, when compared to the Cenozoic anomalies, are, as shown by this tabulation, at a minimum 0.011 greater, at a maximum 0.020 greater, at a mean 0.0155 greater. The difference of the means shown by geodetic measurement was 0.024. The specific gravities seem to have been taken as far apart in limits as is allowable and the assumed mean thickness of sediments as 5,000 ft. beneath the Cenozoic stations is a generous figure; the mean thickness is more likely to be less, rather than greater. The means for the geodetic anomalies as related to geologic formations are perhaps subject to about the same degree of error as the determinations of the anomalies from the specific gravities and thickness. The result, although not of a high order of accuracy, shows that although the range in specific gravities accounts for a considerable part, perhaps one-half or two-thirds, of the relation of anomalies to geologic formations, it can hardly account for the whole.

To find the cause for the remaining portion of the anomaly, two hypotheses may be considered: first, that it is due to a slight regional excess of density extending to a depth of 114 km., the hypothesis favored by Bowie; or, second, that the Archean areas on the average stand higher than the Cenozoic by virtue of resistance to erosion.

The geologic evidence as it is at present understood is against the first hypothesis and in favor of the second. This statement

is based on the view that Archean and Proterozoic areas have tended to be rising elements of the continent. Erosion instead of sedimentation has been dominant in later geologic time, which is the reason why these rocks are now exposed as surface formations. If there is any deep-seated departure of density from the mean this tendency to rise should correspond, however, to a deficiency of density persisting through the geologic ages, extending through much of the zone of compensation and offsetting the more than average surface density. Such a regional deficiency is opposite in character to the excess which is postulated by Bowie as an explanation of the positive anomalies.

Assume then as the next step in the argument that the density of the zone of compensation beneath the pre-Cambrian areas to a depth of 114 km. is the same as under Cenozoic areas except for the outer 5,000 ft., both having a mean density of 2.75 to 2.80, but taken here as 2.67. The outstanding anomaly in that case is due to a longer mean column for the pre-Cambrian areas and consequently greater mass above the level of complete compensation. If the mean radius of these longer pre-Cambrian and shorter Cenozoic columns is as great as 166.7 km., then the unit excess or deficiency of mass of 100 ft. at density 2.67 when spread over these columns will correspond to an anomaly of 0.0024. If the mean effective areas of the pre-Cambrian and Cenozoic formations affecting individual stations are less, the unit mass will give a smaller unit anomaly. If the mean effective areas are greater, the unit anomaly will not, however, rise above 0.0035. Assume then in conclusion a mean radius of 166.7 km., an anomaly of 0.0024 dyne as resulting from 100 ft. of added mass of mean density, and the outstanding anomaly not accounted for by the surficial densities but due to an outstanding difference in volume as between 0.008 and 0.012. These figures correspond to a differential mean elevation of 330 to 500 feet of the pre-Cambrian above the Cenozoic, due to erosion. To physiographers such a conclusion will seem quite in accord with the geologic evidence testifying to the resistance of pre-Cambrian formations.

The character of the Archean and Proterozoic anomalies enters into the problem of crustal rigidity in the following way. If there

were local and close compensation, then as erosion removed the softer surrounding rocks there should be isostatic upwarping of such areas of denudation and relative downwarping of the uneroded crystalline areas. Such warping of the Mohawk, St. Lawrence, and Champlain valleys with respect to the Adirondacks has not been noted, though the problem from the standpoint of field evidence has not been fully studied. The physiographic evidence that residual mountain masses known as monadnocks or unakas have not been shown, however, to be marked by local downwarping and, on the contrary, certainly stand in relief due to circumdenudation, combines with the geodetic evidence of the average excess of gravity for the resistant areas of pre-Cambrian formations, to suggest effective rigidity against the stresses produced by erosion. The evidence, however, as developed thus far from the geodetic standpoint shows that there are more important factors than that of the surface geologic formation, since the larger anomalies are much greater than these figures which have been discussed and hold but little relation to either relief or surface geology. In fact Hayford and Bowie do not find any discoverable relation between the anomalies in general and the topography.

It is thought by the writer, however, that if stations were located especially to test the intensity of gravity over various broad plateaus remaining by circumdenudation and the intensity compared with that over adjacent broad areas of lower level, the mean differential anomalies due to the surface excess of mass in the plateau over the lowlands would rise to a larger figure than the 0.008 to 0.012 dyne which has remained to be explained in the present discussion. These figures are low because certain pre-Cambrian areas, like those in the vicinity of Baltimore and Washington, have been lowered by prolonged denudation and do not stand markedly above the level of younger formations. Furthermore, the tendency of broad pre-Cambrian areas to stand above sea-level is very probably of an isostatic nature. This implies under such areas a slightly lower mean density to the whole zone of compensation which would diminish the anomaly due to the surface elevation. In individual areas of 100 to 200 km. radius, however, such a relation of positive anomaly to pre-Cambrian

formations and plateaus of circumdenudation may not be found, since it is clear that the anomaly from this cause may be much more than neutralized by other causes. A large number of stations covering broad areas would therefore be required adequately to eliminate these other influences from the means.

LARGE OUTSTANDING ANOMALIES NOT RELATED TO GEOLOGY OR TOPOGRAPHY

In Fig. 5, of Part II, the anomalies are shown for all stations in the United States. It is seen that they possess an areal gradation in magnitude which permits the drawing of anomaly contours. The excessive anomalies of both signs cover oval areas in various parts of the country and show a common disregard of physiographic provinces, structural provinces, and geologic formations. Looking at Fig. 5, one cannot see in either the distribution of anomalies or trends of contours a reflection of Atlantic Coastal Plain, or Appalachian Mountains, or Mississippi Valley.

Typical examples of the lack of necessary relation of the large anomalies to geologic formations are seen in the following tabulation:

TABLE XVI

No.	Station	Geologic Formation	Anomaly
123	Albany, N.Y.	Cambro-Ordovician	−0.043
74	St. Paul, Minn.	Cambro-Ordovician	+0.059
96	Mena, Ark.	Pennsylvanian	−0.052
101	Helenwood, Tenn.	Pennsylvanian	+0.040
53, 56	Seattle, Wash.	Quaternary	−0.093
112	Olympia, Wash.	Quaternary	+0.033

The lack of relation of these anomalies to topography is equally striking. It is clear then that internal conditions in the crust, not expressed on its surface, must be the principal cause of these larger departures from isostasy. The large anomalies show their relationship to internal causes most clearly, but the smaller anomalies may also by analogy be ascribed in part to such hidden causes. The results, however, of surface activities—circumdenudation, sedimentation, tangential pressure, or extravasation—must show in large ratio over regions where the internal variations from uniform density are small; but over the greater part of the United States

the distribution of anomalies appears to depend more upon the internal than upon the external departures from regional uniformity and complete isostasy. The internal heterogeneities of mass are therefore presumably greater than the shiftings of mass due to external activities.

CRITERIA FOR SEPARATING VERTICALLY IRREGULAR COMPENSATION FROM REGIONALLY INCOMPLETE COMPENSATION

Suppose the topography smoothed out to a mean level over areas as large as the limits for regional isostasy. The deflection residuals and gravity anomalies would then be due to one or more of three internal causes; first, vertically irregular or laterally displaced compensation; second, regionally incomplete compensation above the bottom of the zone of compensation because of the effective rigidity of the crust above that level; third, regionally incomplete compensation above a certain level because the zone of compensation may be deeper in places, transferring stresses into a deeper rigid earth. The existence of a general approach toward compensation and away from absolute rigidity suggests that the last is not so important as the first two causes. Under this section then will be considered these two causes, their effects upon the deflections of the vertical and the intensity of gravity, with the purpose of drawing criteria by which the action of the two causes may be recognized and separated. To do this it will be necessary to discuss here to some extent the theory of the attraction of underground masses upon stations at the surface of the earth. It has been shown that balanced irregularities in the vertical distribution of densities through the zone of compensation could give pronounced anomalies without disturbing the isostatic equilibrium at the bottom of the zone, since the total weight of the column could still be normal. To show the effect of such balanced irregularities upon a point outside of the column:

Take a vertical line and a horizontal line which intersect. The masses whose effects are to be investigated will be distributed on the vertical line. The effects are to be determined for points on the horizontal line. To express the trigonometric relations between any point on the vertical and any point on the horizontal line, let a point on the vertical line at depth D be defined as at a vertical

angle θ below a point on the horizontal line; the latter to be defined as at distance R from the intersection.

Let the gravitative attraction of unit masses along this vertical line upon any other point either in or outside of this line be represented by F. The horizontal component will be the force producing deflection of the vertical and may be represented by Fh. The vertical component will give the acceleration of gravity due to the unit mass and may be represented by Fv. Taking the unit mass such that the constants will have a value of unity, the following relations are deduced:

Attraction of unit mass at depth D, upon a point at R:

$$Fh=\frac{\cos^3\theta}{R^2}$$

$$Fv=\frac{\tan\theta\cos^3\theta}{R^2}$$

For the intersection point,

$$R \text{ and } \theta=O \text{ and}$$

$$Fh=0$$

$$Fv=\frac{1}{D^2}$$

Let the depth of the zone of compensation, 114 km., be taken as unit distance, 1.00, and for purposes of discussion let points I, II, III, IV be located on a vertical line at depth of 0.25, 0.50, 0.75, and 1.00 as shown on Fig. 6. Solving the equations for these points and for various values of R gives the following tabulation:

TABLE XVII

TABLE OF RELATIVE ATTRACTIONS

(Not in dynes per gram)

Attraction by Unit Masses at			Attraction at Stations for Various Values of R									
No.	Depth	Angle below $R=1.00$	$R=0$		$R=0.25$		$R=0.50$		$R=1.00$		$R=2.00$	
			Fh	Fv	Fh	Fv	Fh	Fv	Fh	Fv	Fh	Fv
0..........	0	0	0	0	16.00	0	4.00	0	1.00	0	0.25	0
I..........	0.25	14°02′	0	16.00	5.60	5.60	2.88	1.44	0.91	0.23	0.24	0.03
II..........	0.50	26°34′	0	4.00	1.44	2.88	1.40	1.40	0.72	0.36	0.23	0.06
III..........	0.75	36°52′	0	1.78	0.51	1.52	0.68	1.04	0.51	0.38	0.21	0.08
IV..........	1.00	45°00′	0	1.00	0.21	0.91	0.36	0.72	0.35	0.35	0.18	0.09

Fig. 6 shows the curves for $R=1$. For any other value of R the curves would be the same in form, but the scales of ordinates and abscissas would be changed. These curves may be used therefore in a general way.

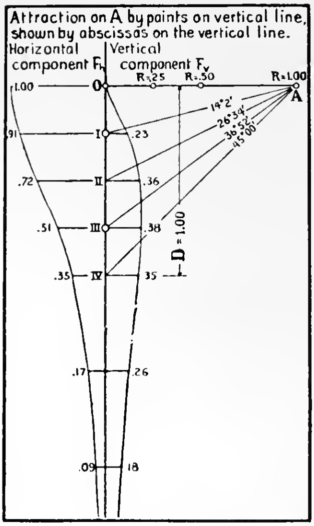

FIG. 6

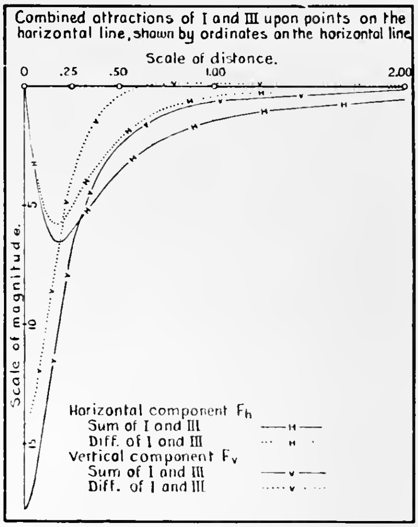

FIG. 7

FIG. 6.—Curves showing relative attraction of all points on the vertical line upon a point at distance $R=1$.

FIG. 7.—Combined attractions upon all points on the surface by unit masses of like and unlike signs at I and III of Fig. 6.

The table shows that if unit masses at II and III have the same sign the *horizontal component*, *Fh*, for the *sum* of their attractions at $0.25R$ will be 1.95, at R it will be 1.23, which is 63 per cent of the value at $0.25R$. If the unit masses have unlike signs the *horizontal component of their difference* at $0.25R$ will be 0.93, at R it will be 0.21, which is but 23 per cent of the value at $0.25R$. The *vertical component*, *Fv*, due to the *sum* of the masses at $0.25R$ is 4.40; at R is 0.74. The *vertical component* due to the *difference* at $0.25R$ is 1.35; at R is 0.02 and of opposite sign. It is noticed that the gravity anomaly diminishes rapidly with increasing horizontal distance from these two masses and passes through zero. The deflection of the vertical first increases sharply and

then diminishes, but less rapidly than the gravity anomaly. It is important to notice that in both cases the total influence due to masses of *opposite* sign diminishes much more rapidly, and where their distance apart is 0.25 their influence is small at distance R and negligible at $2R$. This gives a means of determining whether, in the crust, anomalies and deflections are due to regional departures from isostasy or to balanced irregularities in density without absence of isostasy at the base of the zone.

To give a further illustration of balanced departures in density spread over a greater vertical distance, and representing in that way perhaps a more average case, assume that an excess or deficiency equivalent to a unit mass is at depth 0.25 and another at depth 0.75. The following tabulation shows their influence upon the surface of the earth at increasing horizontal distances.

TABLE XVIII

ATTRACTION BY UNIT MASSES AT I AND III UPON POINTS ON THE HORIZONTAL LINE

Component	Position and Sign of Mass	Horizontal Distance on Surface of Earth from Vertical Line					
		0	0.25	0.50	1.00	2.00	4.00
Fh	− I, −III	0	−6.11	−3.56	−1.42	−0.55	−0.121
Fh	− I, +III	0	−5.09	−2.20	−0.40	−0.03	−0.003
Fv	− I, −III	−17.78	−7.12	−2.48	−0.61	−0.11	−0.015
Fv	− I, +III	−14.22	−4.08	−0.40	+0.15	+0.05	+0.007

The data in this table are represented by the curves of Fig. 7. It shows that for this arrangement of masses the influence on the surface falls off rapidly at a horizontal distance between 0.25 and 0.75, which are also the vertical depths to I and III. When the masses are of opposite sign the anomaly passes through zero at a horizontal distance of about 0.6, and the deflection force for opposite sign decreases to half the value of the sum at about 0.75. The ratio between the effects of like and unlike masses becomes more marked the greater the distance of the point, although the actual magnitudes of the forces decrease.

Now assume the unit masses at I and III to be parts of masses of like density extending to the left of o to a distance N. Consider the aggregate effect upon a given point, as that at 0.50, or in general at point R. The effect of each unit at distance x to the left of o upon the point at 0.50 will be measured by an ordinate at a distance x to the right of 0.50. This will give the same aggregate result as concentrating the masses at o and summing up the area of the curve to the right of the point at 0.50 to a distance of $0.50+N$. Stated in general terms, masses at depths I and III extending linearly to distance N to the left of o will have an aggregate effect upon a point R equal to the area of the curve between R and $R+N$.

As to the aggregate effect on Fv, the gravity anomaly: If the two sheets are of negative density, it is seen that the result will be an increased negative anomaly over the effect of the separate unit masses. If the lower mass is, however, of positive density, the result for ordinarily limited sheets will be a change between o and 0.50 from a large negative to a small positive anomaly. This may be compared with the effects of other possible distributions of mass upon the gravity anomaly.

If the anomaly due to the *adjacent* departure from uniform distribution is of the mean value or greater, the *more distant* abnormal masses will have but relatively small influence. This is because the higher anomalies, with the exception of Seattle, are but two or three times the mean. Further, in a zone of large radius there are a greater number of positive and negative departures. Their aggregate effect, according to the laws of chance distribution would increase but slowly and this effect is diminished by distance according to the formula $\frac{Fv = \tan\theta \cos^3\theta}{R^2}$.

A reversal from a large anomaly of one sign to a *large* anomaly of opposite sign, rather than a *small* one of opposite sign, marks then in general a passage from an area of excess or deficiency of mass to the opposite. A gradual change in the anomaly is the reflection of a change in the subsurface abnormalities nearly as gradual. If the areal variations show that the passages of the anomaly through zero are not frequent, they go to show that limited notable irregularities of density of opposite sign in the

same column are rare. Furthermore, it has been shown under the topic "The Variable Rate of Compensation upon Gravity Anomalies" that a variable distribution of balanced densities has more effect if in areas of between 100 and 200 km. radius and has but little effect on anomalies if the balanced densities extend over much larger areas.

As to the aggregate effects produced upon *Fh*, giving deflection residuals, by these sheets I and III: If the sheets have like sign the deflection force, as shown in Fig. 7, will die out somewhat gradually and extend to considerable distances. If they have unlike sign the deflection force will fall off sharply between 0.25 and 1.00. If, however, the abnormalities of density should disappear gradually, that is, if the sheets did not terminate sharply at 0, this rate of falling off would be slower. Reversals of sign of the *deflection residuals* would require areal, not vertical, irregularities of mass. They could not take place as an effect of distance from a single mass or of two masses of unlike sign and vertically over each other. Where sharp reversals of sign take place in the deflection residuals the presence of areally contiguous areas of unlike departures in mass is shown. A mere difference in magnitude of excess of mass but of the same sign may, however, produce changes in the sign of the deflection residuals. In the irregular areal distribution of abnormal masses not balanced by being over each other, the deflection areas of like sign would thus tend to be smaller than the anomaly areas of like sign. A gradual fading-out of the deflection residuals would be the mark of gradual fading-out of the abnormal mass or the increasing influence of distant masses.

Various special combinations of three or more masses could at any one point simulate the relations indicated, but such special relations would not be of common occurrence and could not give a generality of relation of this sort.

There have thus been drawn up a set of criteria by which balanced irregularities within the zone of compensation may be distinguished from regional departures from isostasy. It remains to apply those to the areal distribution of gravity anomalies and deflection residuals as given by Hayford and Bowie. It must be recognized, however, that the stations, although numerous as

compared to previous measurements, are yet very scattered for the precise application of these tests and can at best give but qualitative results. It is thought, nevertheless, that the general nature of the answer is determinative.

GRAVITY ANOMALIES CAUSED LARGELY BY REGIONAL DEPARTURES FROM ISOSTASY

The first question is: To what degree do the areas of excess (or deficiency) of mass as indicated by gravity anomalies coincide with areas of excess (or deficiency) as shown by the deflection residuals? In Fig. 5[1] there are indicated a number of ovals shown in dot-and-dash outline and marked + or −. These are the definitely bounded areas of excess or deficiency of mass indicated by the deflection residuals. The entire surface of the crust must be constituted of such areas, but only a few are surrounded by sufficient observations to permit a boundary to be drawn at present. Even this boundary must not be regarded as sharply definite. Beside these ovals there are shown in illustrations 5 and 6, Hayford, 1909, areas of residuals characterized by like sign, referred to in the present paper as "areas of grouped residuals." They are not definitely bounded on all sides and are not shown in Fig. 5 of this article. The areas of grouped residuals show the intercepts across areas of like sign, but at least two intercepts at an angle to each other are necessary to define well the limits of the area of which they are a part. As the deflection stations are situated largely in lines or zones across the country and not surrounding the areas of like sign, it is seen why the boundaries of relatively few areas are well determined. In so far, however, as the relations of the areas of positive and negative anomaly to positive and negative deflections of the vertical are apparent, Hayford and Bowie state: "The gravity anomalies corroborate the evidence given by the deflections. In no important case are the anomalies and deflections contradictory."[2]

It is seen by inspection of the illustrations by Hayford, and also by the discussion in Part II of this article, that the areas of

[1] P. 153, Part II.

[2] Hayford and Bowie, 1912, p. 112.

like sign of deflection residuals are more sharply bounded and smaller in size than the areas of like sign of gravity anomalies. The latter occur commonly in areas so broad that a vertically balanced irregularity in the distribution of density would have but little effect. Yet the large gravity anomalies occur in the midst of such large areas, as shown on Fig. 5. There are, furthermore, few sharp reversals of sign of the gravity anomalies save those at different elevations in mountainous regions and these are explained by the presence of regional compensation. There are, on the contrary, many sharp reversals of the deflection residuals.

It is to be concluded, therefore, that, although some degree of balancing of irregularities in the same column no doubt exists, this is not a common or controlling explanation of the anomalies and residuals. They are overshadowed by a distribution which points, on the contrary, to regional departures from isostasy by regional excesses or defects in density.

In the location of stations, the deflection observations are arranged at relatively close intervals and in linear zones, owing to the necessity of triangulation. They give the most information as to the size of areas of relative excess and defect. But two areas of relative excess and defect may both be in absolute excess or absolute defect. The gravity stations are more widely scattered. The local variations are in consequence poorly defined, but the limits of absolute excess and defect of mass are determined with more accuracy. They appear to show that areas as large as 1,000 by 2,000 km., 620 by 1,240 miles, may depart in one direction from isostasy, but only to a moderate amount. It is seen from Fig. 5 that between Florida and a line drawn from Lake Superior to the Rio Grande the broad areas of less than mean anomaly are negative. From this line a great positive area extends to the northwest. The quarter of the United States bordering the Pacific Ocean is, however, another great region of negative anomalies. Upon these broad regions of mean anomaly or less are superposed smaller and better-defined areas of more than mean anomaly, negative and positive areas occurring in the same broad region. These smaller areas are inclosed by the 0.020 anomaly contour. They commonly range from 300 to 400 km. across, 200 to 250 miles, but the maxima

which reach above 0.040 are much smaller. The limits of regional isostasy appear then to vary with the amount of the load. Well-defined areas 200 to 250 miles in breadth may stand vertically 800 to 1,600 feet on the average from the level, giving isostatic equilibrium, and their central portions reach still higher values. They represent the limits of regional isostasy discussed in an earlier part. But these are superposed on broader areas which may extend for a thousand miles or more and lie as much as 400 to 800 feet either above or below the level for equilibrium. Stresses given by loads of this order are then not restricted in area to the limits set for higher values.

The size of the areas of intenser stress reveal the capacity to which the earth can carry mountain ranges uncompensated by isostasy. The size of the areas of weaker stress shows the capacity of a considerable portion of a continent to lie quiescent while the surface agencies carry forward their leveling work. This is the present state of this particular continent after a geologic period of world-wide notable vertical movement and adjustment. It is not likely, therefore, that these loads measure the maximum stress-carrying capacity of the earth. They may be more in the nature of residual stresses which the earth can hold through periods of discharge of stress. East of the Cordillera there has been but little local differential movement and these areas have lain in crustal quiet for long geologic ages, being subject only to broad and uniform crustal warping of moderate amount. It is to be presumed, therefore, that the strains which exist in such regions by virtue of the regional departures from isostasy are of ancient date and well within the limits of crustal strength.

It would seem probable for such conditions, from the standpoint of mechanics, that the zone of compensation is not sharply limited, with its implication of marked lowering of rigidity at its base; nor the distribution of compensation uniform to the base. It seems more probable that the abnormalities of density and the resultant strains should fade out through a considerable depth more after the manner suggested by Chamberlin.

[*To be continued*]

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THE STRENGTH OF THE EARTH'S CRUST

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PART IV. HETEROGENEITY AND RIGIDITY OF THE CRUST AS MEASURED BY DEPARTURES FROM ISOSTASY

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INTRODUCTION AND SUMMARY

In Part I were examined certain geologic tests of the strength of the crust; in Part II the geodetic evidence in regard to the effective areal limits of rigidity; in Part III the influence of variable vertical distribution of density. All three lines of investigation converge toward showing the rigidity of the outer crust—the zone of isostatic compensation—to be such that very considerable stresses can be carried over areas whose radii range between 100 and 300 km. There arise to be considered next the following problems: first, the variability in depth of compensation and its influence; second,

whether the stresses represented by the incompleteness of isostasy are carried by the rigidity of the outer crust, or are transferred in some measure to the deeper body of the earth; third, the magnitudes of the stresses, measured in terms of loads, which are indicated by the gravity anomalies and deflection residuals.

It is found in answer that under the hypothesis which forms the basis of Hayford's work, that of uniform compensation, complete at a given depth, there are indications, given by comparing different areas, of a great range in the depth of the bottom. Under an assumption which is probably nearer to nature—that is, the hypothesis of a variable and gradually disappearing compensation—there is room for even a greater heterogeneity of the crust and a greater variability in the depth reached by the zone of compensation. But, on the other hand, it is concluded that the zone of compensation, as an outer rigid crust separated from the rigid inner earth by an intervening zone of lowered rigidity, is a reality in earth structure. The stresses due to the heterogeneities of density and relief within and upon this crust appear to be borne by the crust, not by the inner earth. Under the third subject it appears, upon review of the evidence given by the deflection residuals, that these may be interpreted so as to show departures from equilibrium comparable to the results given by the gravity anomalies, instead of the 250 feet which Hayford thought to exist. The two independent lines of geodetic investigation are thus seen to agree and it may be concluded with some confidence that the individual isostatic regions of the United States are on the average between 600 and 900 feet out of equilibrium. Evidence from other parts of the world appears to show, furthermore, that a number of regions exhibit greater departures from isostasy than those observed within the United States. The strain imposed on the crust by the Niger Delta, though large, is apparently not as large as some made known by geodetic measurements.

Thus from various directions of attack the crust is shown to be an earth shell of high rigidity and consequently high elasticity. Geodetic evidence justifies the view, brought forward by geologic evidence, that the delta of the Niger is to be looked upon as supported by the strength of the crust.

VARIABLE OR CONSTANT DEPTH OF COMPENSATION

The Cordilleran region awoke to an era of great orogenic and igneous activity near the beginning of the Tertiary, and, especially in the Neocene, has become broadly elevated into one of the great plateau regions of the world. Large areas like the Colorado plateaus, which since the beginning of the Paleozoic had rested near sea-level, at times beneath and again slightly above, have been lifted many thousands of feet. Block-faulted structures indicate the dominance of vertical forces rather than surficial compression as the cause of these movements. The uplift has not been of the nature of a broad even upwarp, and adjacent regions show great contrasts in elevation. These different surface results of the interior forces suggest differences in elevatory forces at comparatively shallow depths. The region is known to be in a fair degree of isostatic equilibrium notwithstanding the high relief. Davis has shown why these movements cannot be regarded as differential sinkings toward the center of the earth.[1] These features suggest, then, subcrustal decreases in density during the Tertiary as a cause of the broad movements of elevation.

The rising of great bodies of magma to high levels in the zone of isostatic compensation, their irregular distribution, the great quantities of heat and gases which would invade the roofs are suggested by the observed evidences of regional igneous activity at the surface as the probable causes of the changes in density and regional vertical movements. A consequence of such a cause would be a lessened strength of the crust to resist strain, a lessened depth to the zone of isostatic compensation, and a decreased size of the unit areas departing from equilibrium.

The history of the Cenozoic in the Cordillera has repeated the history of other regions at other times, either in the Archean igneous activity or later. The slow conduction of this excess heat from the outer crust, the solidification of the reservoirs of magma, would, in the course of ages, bring about a new rigidity. Upon disturbances of the equilibrium by erosion or compressive forces there would be found a new and greater depth to the zone of compensation.

[1] "Bearing of Physiography upon Suess's Theories Abstract," *Intern. Geog. Cong., 8th Report*, 1905, p. 164; *Amer. Jour. Science* (4), XIX (1905), 265–73.

Where ages of uplift and erosion have followed periods of igneous activity there are revealed great bodies of intrusive rock varying in density from granites at 2.65 to gabbros at 3.0. These great batholiths are of irregular distribution in the crust, both vertically and horizontally. Their abundance increases downward so far as erosion has revealed the evidence. The outer crust of the earth has become vertically and areally heterogeneous by such means and should cause variations and irregularities to an appreciable degree in the distribution of isostatic compensation, as noted under the topic of the influence of variable rate of compensation upon gravity anomalies. Here we note in addition the decreased depth of compensation and decreased rigidity at the time of intrusion.

Hayford notes that the stations classified into geographic groups show as a rule as great contradictions in depths of compensation between adjacent groups as in those which are far apart. This variation between adjacent groups is taken by him as weakening the evidence that there is any real variation in the depth of compensation over the whole area investigated.[1] For the reasons outlined previously, the present writer, influenced by the geologic inferences, does not view such irregularity of distribution as proof that the evidence is weak and conflicting. The strength of the evidence must be judged rather by the nature of the residuals. Hayford points out that the depth of compensation in the West seems on the whole to be somewhat less than in other parts of the United States, though he does not regard it as safe to assert that it does exist. On dividing the whole area into four sections, the minimum sum of the squares of the residuals indicates depths as follows as best satisfying the hypothesis of uniform distribution of compensation:

From all residuals of the central group, 174 km.
From all residuals of the northeastern group, 187 km.
From all residuals of the southeastern group, indeterminate.
From all residuals of the western group, 107 km.[2]

In the 1909 paper Hayford gives a tabulation of the residuals for fourteen geographic groups. The results for the United States as a

[1] 1909, pp. 58, 59.

[2] 1906, pp. 142–46.

whole and for the groups showing the shallowest, deepest, and the most irregular compensation are quoted below.[1] That solution is regarded as nearest the truth which gives the smallest mean value of the squares of the residuals.

TABLE XIX

PROBABLE DEPTHS OF COMPENSATION

Group	No. of Residuals δ	Mean Values of the Squares of the Residuals in Various Groups					Most Probable Depth
		Solution B Infinite Depth	Solution E 162.2 Km.	Solution H 120.9 Km.	Solution G 113.7 Km.	Solution A 0.0 Km.	Km.
United States (all observations)...............	733	146.50	14.05	**13.73**	13.75	25.77	**122.2**
Group 12 (parts of Minn., N.Dak., S.Dak., Neb., Kan.)...............	36	196.57	**7.00**	7.47	7.59	11.46	**305**
Group 5 (Mich., Minn., Wis.)	52	34.97	**23.60**	23.64	23.67	27.53	152
Group 8 (parts of Utah, Nev., Cal.)..........	42	128.97	22.27	18.79	**18.25**	35.78	**66**
United States (residuals multiplied by 1.327 to compare with Group 8)	....	194.40	18.65	**18.23**	**18.25**	34.21	....

It is seen that the mean value of the squares of the residuals in group 12 with most probable depth of 305 km. is considerably less than for the United States as a whole, in part no doubt owing to the moderate relief, yet the differences between the residuals in group 12 for the different solutions is much more pronounced than for the United States as a whole. The number of stations, 36, is large enough so that this can hardly be regarded as accidental. On the contrary, it would appear that for the whole United States the group differences are sufficient to mask in part the accuracy of the mean result of 122 km. and that the depth of compensation within certain groups is more reliable than for the United States as a whole.

In group 8 with most probable depth of 66 km. the mean value of the squares of the residuals is nearly 50 per cent higher than for

[1] 1909, pp. 55–58.

the United States as a whole, a value which may be ascribed to the mountainous relief and the support of individual mountains and ranges by the rigidity of the crust. Nevertheless the residuals for the several solutions fall into a somewhat regular system, and solutions E, H, and G are more sharply differentiated from the most probable one than for the whole United States. They may be compared better with the latter if the residuals for the whole country are multiplied by 1.327 as a factor in order to give the same numerical value under solution G. This is done at the bottom of the table. It would appear from these figures as though the arguments previously given from geologic analysis receive considerable support from the geodetic results and point to a much shallower depth for isostatic compensation in the Great Basin than over certain other portions of the United States. Furthermore, in the examination of the question of local versus regional compensation, it was only the forty mountain stations classified into two groups according to elevation which gave any suggestion that regional compensation to a radial distance of 166.7 km. was not about as probable as more local compensation. In these two lines of geodetic evidence as to limited depth and breadth of compensation there are suggestions therefore which support the geologic inference that the crust of the Cordilleran region may be weaker than over the United States as a whole. On the other hand, the warping or faulting-down of ancient continental areas into marginal sea-bottoms implies an increasing density of the subcrust and therefore possibly an increasing rigidity and strength under such areas. Such a contrast between the Atlantic Ocean bottom and the Great Basin would correspond to the great strength of crust necessary to sustain the delta of the Niger as compared with the moderate rigidity found by Gilbert for the crust beneath extinct Lake Bonneville, located within the limits of group 8.

The regions of shallower compensation in the United States are on the whole marked probably by a higher temperature gradient, the regions of deep compensation by a lower. This is illustrated by the very high gradient of the Comstock mine in Nevada and the very low gradient which is found in the Lake Superior copper mines. The temperature gradient may measure the depth to a zone of low

rigidity, determined by a certain relation of temperature and pressure.

Within an overlying zone of high rigidity, even where it is of uniform depth, the geodetic measurements of the depth of compensation may not, however, show uniformity. If the density is unequally distributed, the compensation of a region may be nearly completed at some depth above the base of the rigid zone, the lower part consisting of rock of mean density and therefore not possessing influence. A region of deep and marked rigidity, if characterized by notable irregularities in the distribution of either density or relief, would show large residuals. A region characterized by more uniform distribution of density and gentle relief would show lower residuals even with the same rigidity. A region with deep compensation would show within the limits of the group lower residuals for the same degree of uniform compensation, than where compensation was at lesser depths, since the attracting masses are spread over a greater distance.

As applications and tests of these principles, it is to be noted that group 5, embracing the Lake Superior region with its low-temperature gradients, has the highest residuals of any group in the United States. Further, the mean values of the least squares for the different solutions show less differentiation than in any other group. These facts suggest irregular distribution of density, high rigidity, and the zone of rigidity may extend below the most probable depth, 152 km., indicated for the limits of compensation. The topographic deflections are only 58 per cent compensated. The contiguous group to the southwest, No. 12, shows the lowest residuals of any group, the separate solutions are sharply differentiated and the depth is the greatest in the United States. On the side of this area, the gravity anomaly at St. Paul, 0.059 dyne per gram, is, next to Seattle, the largest found thus far in the United States. It may be concluded, then, that in this part of the continent, undisturbed by igneous activity or mountain-building since the pre-Cambrian, the depth of the zone of rigidity appears to be very great. The irregularities in residuals in group 5 may date from the Keweenawan period, when enormous masses of basic and therefore heavy magmas were intruded and extruded in the Lake

Superior region. If such be the case it shows the long endurance of strains borne by this part of the earth. In the almost universal epeirogenic movements which marked the close of the Tertiary and opening of the Pleistocene, the Lake Superior basin showed notable downwarping, its bottom being now beneath the level of the sea. It formed a trough which directed the flow of glacial ice. The latter must have scoured it clean but can hardly be ascribed as the cause of the existence of the basin. The crust movements have doubtless been in the direction of relief of stress, but the relief has been but partial; geodetic investigation reveals that the age-long load is yet borne.

DEPARTURES FROM ISOSTASY SUSTAINED BY RIGIDITY IN THE ZONE OF COMPENSATION

It was concluded under the last topic that the rigidity over certain parts of the earth probably carries the zone of possible compensation as deep as 300 km. even under the assumption of uniform rate, an assumption which tends to minimize the depth; whereas in other regions under that hypothesis it is less than 100 km. in depth. This raises the question whether the regional departures from isostasy are carried as strains within the zone of compensation or are transferred in part to the deeper body of the earth. There are reasons for believing that the former is the case, pointing by inference to a zone of markedly diminished rigidity between the rigid lithosphere and still more rigid centrosphere.

The geodetic evidence consists in the large values of the squares of the residuals for solution B, the solution which postulates extreme rigidity and compensation at infinite depth. For the whole United States, as shown in the Table XIX, p. 293, the mean value of the squares of the residuals for solution B is 10.7 times the value for solution H. But for group 12, that for which the most probable depth of compensation is 305 km., the distinction is still greater; solution B showing a mean-square residual 28 times greater than for solution E. Dividing in this way the value for solution B by the value for the most probable solution, and taking the mean for all those groups which indicate a depth of compensation greater than the average for the United States, it is found that the ratio is twice

as great for the groups with deep compensation as for the United States as a whole. That is, the groups with deep compensation, instead of showing a leaning toward solution B show on the contrary more definitely that it is not true. The hypothesis of uniform compensation complete at a certain depth appears to be more nearly true for regions with deep compensation than for shallow compensation. This does not mean, however, a lesser rigidity of the crust for the regions with deep compensation, their high capacity to carry strain being shown by the large gravity anomalies which are found in places within them.

There seems to be no evidence, however, that the zone of diminished rigidity is sharply bounded or is marked by real liquidity. It is doubtless due to the gradual rise of temperature with depth, overcoming within a certain zone the influence of the increasing pressure. Seismologic and tidal evidences show, furthermore, that under stresses of relatively brief duration the earth acts as a unit and as an elastic rigid body. The physical condition of the zone of low rigidity may approach that of a highly viscous fluid, the time element thus entering within these limits as a fundamental factor. This zone is incapable of bearing pronounced strains for long periods in the manner of the zone above. In geologic operations it thus serves to separate the mode of expression of forces generated below from those originating above this level. The former give rise to the great compressive movements in the outer zone, the latter to the vertical movements not determined by tangential compression.

INTERPRETATION OF DEFLECTION RESIDUALS IN TERMS OF MASSES

On p. 59, paper of 1909, Hayford shows that the actual deflections of the vertical average only one-tenth of what they would be if the continent and the portions of the ocean basins which were included in the calculations were both underlain by matter of the same density and the relief sustained wholly by the rigidity of the crust. The effect of the topography calculated on this assumption —that the density is uniform and the larger as well as the smaller features are sustained by rigidity—gives what is known as the topographic deflections. These, as stated above, average ten times

the value of the actually observed deflections. The surface may be regarded, therefore, as nine-tenths compensated by variations of density. The details for the five more significant groups are given below:[1]

TABLE XX

1	2	3	4	5	6	7	8
No.	Area of Group	No. of Stations in the Group	Probable Depth of Uniform Compensation in Kilometers	Mean of Topographic Deflections without Regard to Sign	Mean Residual of Solution H without Regard to Sign	Value in Sixth Column Divided by Value in Fifth Column	Percentage of Completeness of Isostatic Compensation for Solution H
12....	Parts of Minn., N.Dak., S.Dak., Neb., Kan.......	36	305	8.23	2.17	0.26	74
8....	Parts of Utah, Nev., Cal....	42	66	32.23	3.57	.11	89
10....	Cal., southern part........	57	126	65.44	3.91	.06	94
9....	Cal., northern part........	60	176	60.50	2.93	.05	95
14....	Northern Cal., western Ore., and Wash..............	37	84	53.68	3.37	.06	94
	Whole United States......	733	122	30.37	2.91	0.10	90

Group 12 gives the greatest depth for uniform compensation. By using the residual for Solution E, 2.09, the percentage of completeness of compensation would have been 75, a trifle more than for Solution H, but still next to the least perfect in the United States.

Group 8, the Great Basin region, has the lowest depth of compensation but shows about the average approximation to isostatic equilibrium.

Groups 10, 9, 14 comprise the Pacific Coast Ranges. They give the highest topographic deflections of the United States, doubtless on account of the great relief of the ocean basin and continental border, but the actually observed deflections do not differ greatly from group 8 or the mean for the whole United States. The result is that in this mountain region bordering the continent the degree

[1] Taken from pp. 56, 58, 69, and illustration No. 2, Hayford, 1909.

of completeness of compensation is the highest in the United States.

On the basis of the figures for the whole United States Hayford writes: "The average elevation above mean sea-level being about 2,500 feet, this average departure of less than one-tenth from complete compensation corresponds to excesses or deficiencies of mass represented by a stratum only 250 feet (76 meters) thick on an average."[1] It is this last statement, interpreting the deflection in terms of mass, which has meaning to the geologist. It has been widely quoted as perhaps the chief geologic result of the work and yet the writer believes that it is without basis. By an oversight of the author he misinterprets his results. If the present writer is correct in making this statement it should not be taken, however, as a criticism of the mathematical portion of the work.

The sea-level is from the standpoint of the problem of isostatic compensation but little more than a datum surface. Imagine the ocean water to be converted into rock of density 2.7 of the same mass as the water and resting on the present ocean bottom. Every thousand feet of water would be replaced by 380 ft. of rock. Then the sea-level surface after this transmutation is seen to lose all real significance.[2] To show the fallacy of taking this level as a basis for interpreting the departures from compensation in terms of thicknesses, let attention be given to groups 1, 2, 3, 4, 6, 11,[3] which cover the United States east of the Mississippi River. The average departure of these from compensation is 0.11, which on the basis of Hayford's statement means that the surface on the average departs but 275 ft. from the level which would give complete isostatic equilibrium on the hypothesis of uniform distribution of compensation to a depth of 122 km. If, however, this eastern third of the United States be regarded by itself, its average elevation may be assumed as 1,000 ft. (it is probably less). By the same reasoning as Hayford applied to the whole United States, 11 per cent of this is 110 ft. Therefore although the average deflections are slightly

[1] 1909, p. 59.

[2] More accurately, the equivalent rock should be imagined as suspended at the mean depth of the water, but the effect of the difference in level is negligible upon the topographic deflection.

[3] 1909, p. 59.

greater than for the United States as a whole, it would be concluded that for the region east of the Mississippi the departure from the levels giving complete compensation averages not more than 110 ft. instead of the 275 ft. previously stated.

Or, again, imagine a rise of ocean-level so that the average elevation of this part of the continent is reduced to 100 ft. without changing the detail of the topography. The deflections would suffer only small alterations due to the added mass of water. Although the crust remained without change, the same reasoning would then lead to the conclusion that the topography departed on the average but 11 ft. from the levels which would give complete compensation.

In computing the influence of the topographic irregularities and their compensation upon the deflection of the vertical, all the topography was taken into account up to a radial distance of 4,125 km. from each station. This radius is approximately the length of 37° of latitude. It embraces the Pacific Ocean out to the Hawaiian Islands and to ten degrees south of the equator, and the Atlantic Ocean out to the Azores. The relief within this region ranges from −8,340 m. north of Porto Rico to +6,220 m. in Mount McKinley, +6,247 in Chimborazo; a total differential relief of about 14,590 m. About one-half of the topography surrounding the coast stations consists of ocean bed. Even for the stations in Minnesota, farthest removed from the sea, about one-third of the surrounding topography within the limits is deep ocean, but lying at a greater distance and carrying lesser influence. The average depth of the oceans influencing the deflection of the station at mean distance inland may be assumed for purposes of illustration to be about 5,000 meters. This depth of water is equivalent in mass to 1,900 m. of rock of density 2.67, leaving an effective ocean depth of 3,100 m. Add the mean continental elevation of 760 m. to this, and 3,800 to 3,900 m. represents about the effective mean relief between continent and ocean. On coast stations this differential relief has greatest influence. For inland stations the several portions of the continent have proportionately more effect. For the United States as a whole it is this relief of between 3,500 and 4,000 m. between continent and ocean, more than the relief between the major features of the continent, which is nine-tenths compen-

sated by the corresponding variations in crustal density, not the 760 m. which is the average elevation of the United States above sea-level.

It is the belt of Pacific coast stations which measures more closely than other groups the degree of compensation accompanying the continental relief above the ocean bottoms. These stations lie in groups 10, 9, and 14, for which the mean residuals are but 0.06, 0.05, and 0.06 of the mean topographic deflections respectively. These residual deflections indicate that for this coastal zone the departures from complete compensation amount to but 5 or 6 per cent. If the mean effective relief which controls this be assumed as 4,000 m., then the mean departure from equilibrium is represented by a mass 200 to 240 m. thick, approximately between 650 and 800 ft. On the other hand, groups 5 and 12 are those farthest removed from the ocean basins and their deflections are controlled most largely by the internal continental relations. For them the departures from complete isostatic compensation as measured by the ratio of the mean residuals to the computed topographic deflections amount to 42 and 26 per cent. The mass to which this is equivalent may be no greater than the 5 per cent departure on the Pacific coast. These estimates fall into the same order of magnitude as that of the masses represented by the gravity anomalies.

This reconnaissance of the problem is sufficient for present purposes. It is readily seen that even greater difficulties stand in the way of a precise statement regarding the equivalence of mass corresponding to deflections of the vertical than arose in the interpretation of the gravity anomalies. The residual for each observed deflection is the sum of the influences of all the excesses and deficiencies of mass as compared to solution H on all sides of a station. The effect of each unit varies inversely with the square of the distance and directly with the sine of the angle which the line of force makes with the horizontal passing through each station. A combination of the data from the measurements of the intensity of gravity with those of the deflections of the vertical would apparently be necessary to state for each region the equivalence in terms of mass which is implied by the residual at each station.

MAXIMUM LOADS INDICATED BY ANOMALIES

Hayford and Bowie consider that 0.0030 dyne of anomaly may be regarded as equivalent to 100 ft. of rock possessing a density of 2.67. From the previous considerations it would seem that this is probably too high for a mean figure, but may apply to certain areas, especially those with extremely broad boundaries. In other regions 0.003 may be far too high, since it is shown under the topic "Variable or Constant Depth of Compensation" that in certain parts of the United States the depth of the zone of compensation probably goes notably deeper than in other parts and the density may be distributed either nearly uniformly or with considerable irregularity. The greatest depth of compensation indicated for any region is 305 km. A unit thickness of mass uniformly distributed to this depth and to a radius of 166.7 km. would give but 0.0014 of anomaly instead of 0.0024 as given by a depth of 114 km., or 0.0030 as taken by Hayford and Bowie. For general use 0.0024 dyne is perhaps the best value, corresponding to a uniform distribution of a unit excess or defect of mass to a depth of 114 km. and to a radial distance of 166.7 km. For the mean anomaly of 0.018 this would give 750 ft. of elevation as the mean departure of the surface of the United States above or below the position giving isostatic equilibrium, instead of 600, or more exactly, 630 ft. as taken by Bowie. The largest known anomaly in the United States is at Seattle, −0.093. This corresponds to a defect in mass equivalent to a stratum 4,000 ft. thick if the divisor is 0.0024, a stratum 3,200 ft. thick if the divisor is 0.0030. At Olympia, but 50 miles or 80 km. distant, the anomaly is +0.033, corresponding to excesses of mass of 1,375 or 1,100 ft., according to the divisor. The difference of regional load between Olympia and Seattle becomes 5,375 or 4,300 ft.

But these relations of unit thickness of mass to the gravity anomaly are based on the assumption that the excess or deficiency of mass extends to as great a radial distance as 166.7 km. radius. This minimizes the thicknesses or densities needed to account for the anomalies above what would be required for a more local concentration of mass. But an inspection of the distribution of gravity and deflection residuals shows that in many cases the masses

producing the greater disturbances have much smaller size. This is especially striking in the case of the largest negative anomaly in the United States, that at Seattle, only 50 miles from the large positive anomaly at Olympia. The latter is surrounded on all sides by negative anomalies as follows:

DISTANCES FROM OLYMPIA, WASHINGTON

Astoria, Ore.......	76 miles	S.W.	−.013	dyne anomaly
Heppner, Ore......	195 "	S.E.	−.027	" "
Skyhomish, Wash..	84 "	N.E.	−.028	" "
Seattle, Wash.....	50 "	N.N.E.	−.093	" "

The excess of mass which exists in the vicinity of Olympia, above that required for compensation under solution G, must therefore be much less than 166.7 km. (102.5 miles) in radius. The same is doubtless true of that excessive deficiency which exists at Seattle, since the anomaly sinks to less than one-third the value at Skyhomish only 45 miles east, and changes to a large positive anomaly at Olympia, 50 miles south-southwest.

The large positive mass in the vicinity of Olympia must diminish appreciably the effect of the still larger negative mass in the vicinity of Seattle. The latter with the other surrounding negative masses must diminish still more the anomaly due to the positive mass at Olympia. Furthermore, it is highly improbable that the observations at Seattle should happen to be made at the point of really maximum anomaly. Let the very moderate assumption be made then that the abnormal Seattle mass as a unit by itself would give a maximum anomaly of −0.100 dyne. It would doubtless give more. Let limiting assumptions be made as to the dimensions and density of this mass such that the actual volume and density are quite probably embraced somewhere within these limits. Tables XXI[1] and XXII show the results of such assump-

[1] Table XXI is readily derived from Table X, Part III. Take, for example, the cylinder of radius 1,280 meters, depth of 1,000 feet, and density 0.267. Multiply its dimensions by 30 and the volume of each unit portion will be increased by the cube of 30. The attraction of each unit of mass on the given point will vary inversely with the square of the distance and will therefore be diminished by the square of 30. The anomaly will consequently increase directly with the dimensions, provided that the density remains constant. This gives the basis for the calculations in column 2, Table XXI.

tions. In Table XXI the attracting mass is supposed to have the form of a vertical cylinder. With a given anomaly the deficiency

TABLE XXI

VERTICAL CYLINDERS GIVING A NEGATIVE GRAVITY ANOMALY OF 0.100 DYNE AT CENTER OF TOP SURFACE OF CYLINDER

1	2	3	4	5
Diameter	76.8 km.	51.2 km.	102.4 km.	51.2 km.
Depth	9.15 km.	30.5 km.	61.0 km.	61.0 km.
Density	− 0.31	− 0.15	− 0.07	− 0.12
2.80−Density	2.49	2.65	2.73	2.68
Thickness of cylinder of same area and mass, but density 2.67	1,080 m. 3,550 ft.	1,700 m. 5,600 ft.	1,700 m. 5,600 ft.	2,770 m. 9,080 ft.
Anomaly per 100 feet of mass of density 2.67 expanded to depth of cylinder as given in second line	0.0028 dyne	0.0018 dyne	0.0018 dyne	0.0011 dyne

TABLE XXII

SPHERES GIVING A NEGATIVE GRAVITY ANOMALY OF 0.100 DYNE AT POINT VERTICALLY ABOVE ON THE SURFACE OF THE EARTH

1	2	3	4	5
Diameter	50. km.	100. km.	50. km.	100. km.
Depth to center	25. km.	50. km.	32. km.	64. km.
Density	− 0.144	− 0.072	− 0.236	− 0.118
2.80−Density	2.66	2.73	2.56	2.68
Length of polar axis of oblate spheroid of same equatorial section and same mass, but density 2.67	2,700 m. 8,850 ft.	2,700 m. 8,850 ft.	4,420 m. 14,500 ft.	4,420 m. 14,500 ft.
Anomaly per 100 feet of polar axis of mass at density 2.67 if expanded to diameter of sphere	0.0011 dyne	0.0011 dyne	0.0007 dyne	0.0007 dyne

of mass will be least if the cylinder extends from the station downward instead of being at a greater depth. Furthermore, for a given volume and density of cylinder the gravitative force will vary according to the ratio of the depth to the diameter.

Let H = depth
Let $2R$ = diameter
Let F = gravitative force

Then $\pi R^2 H$ = the volume, a constant. To find the ratios of H to R which give maximum attraction for unit mass

Let $R^2 H = 1$ and solve for various values of R the equation

$$F = 2\pi\rho\gamma\left[\frac{(R^3+1)-\sqrt{R^6+1}}{R^2}\right].$$

Several solutions are as follows:

For $H=0.75$, $R=1.15$; $F=(2\pi\rho\gamma)\ 0.523$
$H=1.00$, $R=1.00$; $F=(2\pi\rho\gamma)\ 0.586$
$H=1.50$, $R=0.81$; $F=(2\pi\rho\gamma)\ 0.609$
$H=2.00$, $R=0.71$; $F=(2\pi\rho\gamma)\ 0.586$
$H=4.00$, $R=0.50$; $F=(2\pi\rho\gamma)\ 0.200$

This shows that the gravitative force is a maximum for a cylinder of constant volume and mass in which the depth varies from one-half the diameter to four-thirds the diameter. The force varies but slightly between those limits. The cylinders of columns 3, 5, and 6, of Table XXI, lie within these limits. Thus all the assumptions thus far made favor the minimization of the negative load which produces the Seattle anomaly.

Taking the mean density of the outer part of the lithosphere as 2.80 it is seen that the cylinder of column 2, Table XXI, has a density below that of the lightest rock-making minerals and would require the existence of a molten magma or of abnormal pore space to great depth. It may therefore be eliminated as not probable. Cylinders 3, 4, and 5 show densities within the limits of granite, the lightest of the abyssal igneous rocks. It may be concluded, therefore, that the deficiency of mass, if of approximately cylindrical form, is equivalent to a negative load of between 5,000 and 10,000 feet of rock, extending over a distance of from 50 to 100 km., or a somewhat less local load superposed upon a broad but small regional load of the same sign. The nature of the assumptions has been such that we may conclude with confidence that the Seattle anomaly corresponds to at least 5,000 feet of rock and may reach a considerably higher figure, perhaps 10,000 feet. Furthermore, Hayford's unit mass, extending to the areal limits, 100 feet thick and density 2.67, would here produce an anomaly as low as between 0.0010 and 0.0020 dyne.

Instead of a cylinder suppose the mass which produces the deficiency of gravity to approximate more to the form of a sphere. The results are shown in Table XXII. In columns 2 and 3 the sphere is tangent to the surface, a position diminishing the mass for a given anomaly. In columns 3 and 4 the top of the sphere is 7 and 14 km. deep respectively. The low density of column 4 shows it to be beyond the limiting conditions. The load, though negative in sign, is seen to be equivalent in order of magnitude to the greater volcanic piles; 30 to 60 miles in diameter, 9,000 to 14,000 feet in height for rock of density 2.67. The anomaly produced by the unit mass of 100 feet thickness and density 2.67, considered here as 100 feet of polar diameter for a spheroid of the given horizontal dimensions, ranges between the low values of 0.0007 and 0.0011 dyne.

From a consideration of these two tables it is seen that the large anomalies require either a variation of mass equivalent to as much as 5,000 feet of rock extending over some thousands of square miles or to 10,000 feet of rock, more or less, extending over 1,000 square miles, more or less. These tables determine the order of magnitude, but the data are not sufficient to permit a more accurate solution of the problem.

Thus this detailed examination of the anomalies in the region of Seattle shows that the divisor of 0.0030, as taken by Hayford and Bowie, or 0.0024, as considered here the best for general use, is too high for the more limited areas of high anomaly. The latter may be regarded as made up in part of a regional portion for which the divisor of 0.0024 would be applicable and a local portion for which the divisor is probably not over 0.0015. As a mean value, for the more limited areas of large anomaly the amount due to the unit thickness of 100 feet of rock of density 2.67 should apparently not be taken as over 0.0020 dyne.

In forming conceptions as to the uncompensated vertical stresses existing widely in the earth's crust it is important to know the maximum range of departures from the mean stress as well as the latter. These can be studied well in Fig. 5.[1] The mean of fourteen maxima of defect of gravity is −0.033, the mean for eleven

[1] Fig. 5, p. 153, Part II; also see Hayford and Bowie, pp. 107–8.

areas of excess of gravity is +0.034. With the exception of the Seattle stations with an anomaly of −0.093, none reach a value of 0.060. It is thus seen that the average notable maximum is not far from twice the mean anomaly. Even by using a uniform divisor of 0.0024 or 0.0030 to convert anomalies, regional departures of load amounting to 1,300 or 1,500 ft. over areas of several square degrees are found to be not uncommon. Over smaller areas the loads rise to about three times the mean, and at Seattle to five times the mean. These figures of course do not measure simply the elevations or depressions of uncompensated erosion features. On the contrary, if the hills and valleys be imagined as smoothed out, then the resulting mean surface would be out of isostatic equilibrium in the same direction over distances amounting frequently to hundreds of kilometers and attaining maximum departures too low or too high over smaller areas by these figures.

But an inspection of the contour map of gravity anomalies (Fig. 5) shows that the large anomalies, those of 0.040 dyne or above, are all located by Hayford and Bowie as centers of maximum anomaly, though the nearest adjacent stations average as much as 100 miles distant. Between the widely spaced stations, the anomaly gradients are gentle. But where the stations form a series closer together, as that from the city of Washington to New York City, the gradients are seen to be steeper and more irregular. It is to be presumed that a further multiplication of stations would show increased complexity over the whole country and reveal maxima higher than those now recorded. The value of the mean anomaly without regard to sign should furthermore increase somewhat through the discovery of additional areas of maximum value. Areas of regional positive or negative anomaly would persist in something of their present size, but within broad areas of anomaly of one sign should be discovered smaller areas of opposite sign which are now unknown. Upon the completion of such a detailed survey the high anomaly of Seattle would not appear so exceptional as it does at present.

The chart of the residuals of Solution H[1] shows within the larger areas of like deflection of the vertical many large and sharp

[1] Illustration No. 3, Hayford, Supplementary Paper, Bowie, Illustration No. 5.

variations in value and in direction. The resultants of the plotted arrows point toward the centers of exceptional mass and their rapid changes in value and direction point toward the existence of many comparatively shallow masses. The epicentral points above such masses are those where the gravity anomaly, Fv, is at a maximum. If a hidden mass may be regarded as approaching a spherical form and has its center at depth D, the following relations exist between the value of the gravity anomaly and the distance x from the epicenter:

$$
\begin{aligned}
Fv &= \text{Maximum for } x = 0.00\,D \\
Fv &= .75 \text{ max. } \quad \text{"} \quad x = 0.46\,D \\
Fv &= .50 \text{ max. } \quad \text{"} \quad x = 0.77\,D \\
Fv &= .25 \text{ max. } \quad \text{"} \quad x = 1.23\,D
\end{aligned}
$$

If, for example, an approximately spherical mass has its center at a depth of 32 km., .005 of the earth's radius, the anomaly Fv will fall to half-value at a distance of 25 km. from the epicenter. If the center is 64 km. deep, the anomaly will fall to half-value at 50 km. from the epicenter. Between stations located 100 km. apart by far the greater number of real maxima would be missed, and in so far as they depended upon masses in the upper half of the zone of compensation the indicated maxima would at most places be less than one-half the real maxima.

The stresses acting within the crust owing to excesses or deficiencies of mass are not so concentrated and therefore not quite so great as if those abnormalities of mass existed as surface loads of rock of density 2.67 in the manner imagined for the interpretation of anomalies.[1] Nevertheless to gain a conception of the meaning of the gravity anomalies, imagine the present compensated topography to be smoothed out to sea-level and the variations of mass away from isostatic equilibrium to become variations of volume upon its surface. The anomaly contours will then become topography contours, the line of zero anomaly will become the datum plane. The values in mass to be assigned to the successive anomaly contours can only be given in mean figures. It has been shown however in Part III that balanced vertical irregularities of density

[1] The relations of mass and its distribution to the resulting stresses will be considered in a later part.

do not play a large part in causing gravity anomalies. It will be shown later that neither can nucleal heterogeneity below the 200 to 300 km. level of isostatic compensation account for a large part. The anomalies represent in greater part real departures from isostasy and, as shown in this section, the limited areas of high anomaly are to be interpreted as implying on the whole a local load in higher ratio to anomaly than do the broad areas of anomaly. The average relation thought to exist is shown then in the following table:

TABLE XXIII

AN ESTIMATED AVERAGE RELATION OF ANOMALY CONTOURS TO CONTOURS OF EQUIVALENT ROCK MASSES OF DENSITY 2.67

Anomaly Contour, Positive or Negative	Assumed Divisor for 100 Feet of Rock upon a Level Surface	Equivalent Contour in Feet, Positive or Negative
.020	.0025	800
.040	.0023	1700
.060	.0020	3000
.080	.0018	4500
.100	.0016	6300

Upon conversion of a detailed anomaly map, if such existed, into the equivalent topographic contour map by means of such ratios as those given in Table XXIII, the whole United States with its compensated topography previously smoothed out to sea-level would be reconverted into a roughly mountainous country with no notable distinction between what are now the central plains and mountainous border regions of the continent. On to broad plateaus or basins upward of 1,000 feet from the mean elevation would be added higher elevations and depressions. The extreme differential relief would probably be in the neighborhood of two miles though the average departure without respect to sign from the mean surface of the geoid would probably be between 800 and 1,000 feet. Though everywhere as irregular as a mountainous country, there would be little or no relief of the mean level of this hypothetical surface above the ocean bottoms and no such broad and high masses as the Cordilleran plateaus would remain within

its limits. The major relief of the continent above the ocean bottoms would be about nine-tenths eliminated and the mean elevation of all areas as great as several hundred miles in width would be reduced to a small figure. This is the effect of isostasy. But within these unit areas which measure the limits of regional compensation would everywhere rise a rolling mountainous surface.

Imagine the hypothetical surface as broad as the United States, concealed from view by an impenetrable envelope of cloud, and aerial explorers to sink a sounding line to this invisible land at 124 places chosen at random. The resulting contour map compiled from these soundings would yield a much smoothed and flattened surface such as is shown in the contour map of gravity anomalies. Many of the soundings taken really on mountain slopes, because they were the highest of those made, might be casually interpreted as located on mountain peaks. The latter, standing sharp and high, would be missed save for an occassional lucky chance of the sounding line.

Interpreted in terms of weights and stresses, it is seen that even the parts of the continent appearing to the eye as plains long in geologic quietude really conceal within them strains as great as those imposed by the weight of mountains. That these great strains have been born for geologic ages, in many localities probably from the Archeozoic, gives a surprising conception of an enduring rigidity and elasticity of the crust wholly at variance with certain current doctrines regarding the weakness of this zone. It is not here found to be a failing structure.

On p. 81 Hayford and Bowie give the new-method anomalies for sixteen stations not in the United States. An abstract is given below of the greater anomalies from that table with the addition of Seattle. The thickness of stratum taken as corresponding to the anomaly is also added. This thickness, if the compensation is uniform with depth, measures the distance by which the earth's surface is out of isostatic equilibrium at those points. A plus sign indicates an excess of mass and a consequent tendency to sink, resisted by rigidity; a minus sign a defect of mass and therefore the existence of an upward strain.

The divisor 0.003 dyne of anomaly, taken as the equivalent

of 100 feet of abnormal mass of density 2.67, is Hayford's figure. As previously discussed it is thought to minimize too much the thickness of equivalent rock. It is given, however, for comparison with the column derived from the use of 0.0024 as a divisor. This is regarded as a better average figure, but for some cases at least, as shown for Seattle, this also may give too low a result.

TABLE XXIV

Number and Name of Station	Elevation in Meters	New-Method Anomaly	Thickness of Equivalent Stratum on Land in Feet	
			0.003 Anomaly =100 feet	0.0024 Anomaly =100 feet
2. Tonga plateau, Hecker, at sea	−2,700	+0.255	+8,500	+10,625
4. Tonga deep, Hecker, at sea	−6,500	− .184	−6,130	− 7,660
9. Mauna Kea, Hawaiian Islands	+3,981	+ .183	+6,130	+ 7,660
10. Hachinohe, Japan	+ 21	+ .110	+3,670	+ 4,590
13. Sorvaagen, Norway	+ 19	+ .146	+4,870	+ 6,090
Seattle, United States	....	−0.095	−3,170	− 3,960

It is seen that the excesses of mass indicated for Mauna Kea and at Sorvaagen are each comparable in equivalent thickness and extent to the maximum thickness of the Niger Delta if measured by rock upon land, 5,450 feet. The departures from equilibrium at Hachinohe, Japan, and Seattle, United States, are comparable in thickness and area to the burden of the Nile Delta, the weight in air of 3,600 to 4,200 ft. of rock. In weight as in area, therefore, these deltas are seen to impose burdens on the crust no greater than are found, by means of geodetic observations, to exist in certain other regions where geologic evidence had not revealed them. The accuracy of Hecker's method for determining the intensity of gravity at sea has been called into question by Bauer[1] so that, until

[1] "On Gravity Determinations at Sea," *Amer. Jour. Science* (4), XXXI (1911), 1–18. "Hecker's Remarks on Ocean Gravity," *Amer. Jour. Science* (4), XXXIII (1912), 245, 248.

this question is settled by geodesists, equal weight should perhaps not be attached to the figures given for the departures from isostasy shown over the Tonga plateau and Tonga deep. Neither is the area of these departures known, though the areas of the plateau and deep are large. These regions are seen, however, to indicate considerably higher departures from isostasy than the measurements determined from the deltas of the Nile and Niger. The latter, therefore, perhaps do not measure the full strength of the crust.

Major H. L. Crosthwait has applied Hayford's methods to the investigation of isostasy in India.[1] The residuals of the deflections of the vertical serve as a measure of the degree of compensation existing in the United States as compared in India and are as follows:

United States of America

Group S.E., mean residual	− 0″74
Group N.E., mean residual	− 1.04
Group Central, mean residual	− 1.66
Group W., mean residual	− 4.02

India

Region No. 1, Himalaya Mountains, mean residual	−16″
Region No. 2, Plains at foot of Himalaya Mts., mean residual	− 2.
Region No. 3, N.E., mean residual	+ 8.
Region No. 4, Central, mean residual	+ 5.
Region No. 5, N.W., mean residual	+ 4.
Region No. 7, W., mean residual	− 3.
Region No. 8, E., mean residual	− 2.
Region No. 9, S., mean residual	+ 1.

It is seen that the residuals average several times as great in India as in the United States, which leads him to conclude that "Speaking generally it would appear that isostatic conditions are much more nearly realized in America than in India, i.e., if we are to take the smallness of the residuals as an indication of the completeness of isostatic compensation."[2] Colonel Burrard, utilizing the Hayfordian computations, points out the existence of zones

[1] *Professional Paper No. 13, Survey of India*, 1912.

[2] *Op. cit.*, p. 4.

in India where the deflections of the plumb-line are actually in opposition to the directions called for by isostasy.[1] The major elements of the relief, the Himalayas, the plateau of India, and the surrounding ocean basins are of course largely compensated, but these figures show that in detail the hypothesis of complete isostasy is very far from the truth. Crosthwait suggests that the explanation for the difference between the United States and India probably lies in the magnitude of the recent upheavals of the crust in that part of the globe. Nevertheless such upheavals cannot exceed the strength of the crust, and in India, therefore, perhaps may be better observed than in the United States the maximum strains which the earth is competent to endure.

It may be concluded, therefore, that the convergence of geodetic evidence shows the crust to be competent to sustain loads measured by the weight of several thousand feet of rock extending over circular areas some tens of thousands of square miles in area. This is a measure of crustal strength twenty, fifty, or even a hundred fold greater than that advanced in recent years by the leading champions of high isostasy.

FURTHER GEODETIC WORK NEEDED FOR GEOLOGIC PROBLEMS

It has been the intention in the preceding analysis to show two things: first, that the data set forth by Hayford and Bowie are of great value to geology and establish new methods of research, but, second, that the difficulties inherent in the observations and their mathematical treatment, and the fewness of the stations in comparison with the heterogeneity of the earth, are such that the conclusions from the geologic study of deltas in the first part of this paper are as convincing and perhaps as accurate as the present results of the geodetic studies. The latter, however, opens for the whole earth a field of investigation which the geologic evidence covers very locally and imperfectly, a world-wide field which should be pursued for the geologic as much as for the geodetic bearings.

By means of the divining rods of pendulum and plumb-line the heterogeneities of mass and the loci of strain in the outer crust of

[1] "On the Origin of the Himalaya Mountains," *Professional Paper No. 12, Survey of India*, 1912.

the earth should be sought out and measured in detail. For this work it would seem that many new stations would have to be established; in groups so as to reduce the errors of each locality; in sets so as to attack particular phases of the problems. For example, it would appear that gravity stations should be located in pairs close together and of as great a difference in elevation as possible. Certain stations should be located within areas of plateaus spared by circumdenudation, such as the Cumberland and Allegheny plateaus; others should be located in the broad erosion basins. Deflection stations should be located on the lines separating regions of erosion from those of circumdenudation, and also on the lines separating areas of upwarp from those of downwarp. A network should inclose, finally, all centers of marked gravity anomaly or topographic deflection. Such an increase in the number of stations would permit the introduction of simple hypotheses of variable depth and rate and regional limits of compensation. But such an extensive program is within the reach only of some research institution. It needs the co-operation of geologists and geodesists. The location of stations with respect to surface features and their geologic history should be controlled by the geologist. The density of the rocks to the limits exposed by the structure should also be determined by him. The geodesist, on the other hand, should seek out the hidden heterogeneities in the crust and guide the details of the work.

[*To be continued*]

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THE STRENGTH OF THE EARTH'S CRUST

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PART V. THE DEPTH OF MASSES PRODUCING GRAVITY ANOMALIES AND DEFLECTION RESIDUALS

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SECTION A[1]

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INTRODUCTION AND SUMMARY

The fact that the observed deflections of the vertical are on the average only one-tenth as large as the computed effects of the topographic relief, computed on the assumption of uniform density throughout each earth shell, shows that the densities of the crust

[1] Section B of Part V, on the applications of the criteria to determine the limiting depths, forms, and masses of the excesses and defects of density, will be published in the following number of this *Journal*.

are not uniform, but in a broad way are balanced against the relief. This is the proof that a condition prevails of approximate regional isostasy. As the relief of the globe is highly variable, the densities in the lithosphere are therefore within certain limits also highly variable. But on the other hand, the existence of gravity anomalies and deflection residuals indicates that the variations in density are not completely in accord with the demands of the hypothesis which postulates local compensation of the topography uniformly distributed to a uniform depth, nor apparently with any other simple hypothesis. These quantities measure the differences between the hypothesis and the facts of nature. Let the density variations beyond those required to balance the topography vertically above them be called the outstanding excesses or defects of density and the masses which they represent be called the outstanding masses.

It is fundamental to the problems of the strength of the crust, and a system of geologic dynamics in accord with that strength, to determine the depth, form, and weight of these outstanding masses. Do they belong to the centrosphere or to the lithosphere? As Gilbert has noted, if they are to be referred to the centrosphere they do not imply any imperfection of isostasy nor any competence for stress within the crust. Or, if they exist in the lithosphere, the zone of compensation, but are balanced vertically in the same column by other masses of opposite sign, this arrangement will produce local strains within the crust but not tend to flex the crust as a whole. Neither in this case, therefore, would they measure departures from perfect isostasy. As following questions, are the imperfections of isostasy small and local, and the residuals and anomalies the summation of many scattered effects? Or, on the contrary, are there notable regional departures from the conditions of solid flotation which measure a very appreciable rigidity of the crust? If so, to what extent are these regional outstanding masses related to the mountains, valleys, and deltas in process of evolution under the present cycle of surface activities; producing a progressive unbalancing possibly being slowly restored toward balance by a viscous undertow? To what extent are the departures from isostasy due to variable composition and density of igneous intrusions dating back to earlier geologic ages, perhaps never in

isostatic balance, and supported permanently and rigidly by the strength of the crust?

These problems have been studied in the last three parts of this article by means of the evidence presented by Hayford and Bowie, but that evidence has not been used to solve the depth of the outstanding masses. Yet it is seen that all of the aspects just enumerated are bound up in that factor. It is especially this problem of the depth and the consequent areal extent and mass of the units which produce the residuals of the deflection and gravity observations which is attacked in this part. It is necessary for this investigation to enter into a study of the complex relations between the anomalies and residuals which depend upon the depth and form of masses. It is a subject upon which, so far as the writer is aware, but little has been done, so that about half of this chapter, published as Section A, consists of a study of these relations preliminary to their application.

For facility of mathematical treatment the individual outstanding masses must be regarded as equivalent to spheres, spheroids, or cylindrical disks, either as units or as aggregates. If only the epicenter (the point on the surface vertically above the center) of the disturbing mass is determinable and the deflections at two or three points on one side of it, then the mass may be most simply interpreted as a sphere; since the mass of a sphere acts as if concentrated at its center. With fuller observations a close approximation to the depth of the mass and a less close approximation to its form and density may be made. The first problem then is to determine the epicenter of the outstanding mass and its depth, using for this purpose the nature of the anomalies and residuals. For a sphere beneath a plane surface it is shown that the value of the maximum gravity anomaly at the surface is 2.6 times the value of the maximum deflection residual, both being measured in the same units of force. The former occurs vertically over the abnormal spherical mass, that is, at the epicenter. The latter occurs at a horizontal distance from the epicenter equal to 70 per cent of the vertical depth to the center of mass. Oblate spheroids and broad cylindrical disks with vertical axes and the same depth of center as the sphere give maximum deflections at greater

distances from the epicenter. The curves of the deflection force for these forms, especially the spheroidal forms, resemble somewhat closely those given by deeper spheres of greater mass. If the outstanding masses are in reality horizontally extended the interpretation of the deflection residuals as due to spherical masses assigns to their centers in consequence too great a depth. If, however, the masses have the forms of vertical prolate spheroids or vertical elongate cylinders, the interpretation as spheres will give too shallow a depth. The ratios of the maximum anomalies to the maximum deflections constitute a criterion to show whether the masses depart from spheres by the spreading-out of their substance in a horizontal plane or along a vertical axis.

In Section B the outstanding masses are shown in some cases to be horizontally extended in form and this is thought for the larger masses to be a rather general relationship; that is, the vertical thickness is much less than the length or breadth. Consequently the interpretation as spheres gives maximum limits to the depth.

A general inspection of the geodetic data as well as a detailed study of a certain test region shows that the smaller disturbing masses have their centers in the outer third of the zone of compensation; that is, within 40 km. of the surface. This result is to be expected, since similar small masses at greater depth would not exert a notable effect because their gravitative force varies inversely with the square of the distance. But evidence of more significance is found in regard to the larger centers of outstanding mass not related to topography. These also are found, in so far as they have been investigated, to be situated in the outer third of the zone of compensation. Yet these masses are capable of showing notable effects to distances of from 100 to 150 km. If they were situated at any depth within the zone of compensation they would, therefore, betray both their existence and their depth. The greater departures from isostasy appear, therefore, to be really absent from the deeper parts of the lithosphere.

Centrospheric heterogeneity, if present, would require greater masses in order to show surface effects. But no such effects are noted. In so far as they may be existent, they are largely masked

by the more important attractions of superficial masses and hidden by the indeterminate nature of much of the present data. Therefore centrospheric heterogeneity is not a hypothesis which can be used to account for the apparent departures from isostasy. It can be at most only a very secondary factor.

In the last topic of Section B is discussed the relation of the depth of outstanding masses to the various hypotheses regarding the distribution of compensation. The hypothesis of local compensation, as is perceived to a certain extent by those who have used it, is in error in supposing that variations in density correspond to every topographic feature and extend uniformly to the bottom of the zone. But these errors, whether they be small or great, are so spread out in depth and their centers of attraction are consequently so far removed from the surface that they have little effect on the geodetic observations. Especially is this true in comparison with those large and concentrated outstanding masses due to batholithic invasion or other causes which are found to exist at moderate depths in the outer crust. The reasons then why the deflection residuals and gravity anomalies appear to show so little relation to local surface relief and larger physiographic provinces are threefold: in part because of a regional compensation; in part because the hypothesis of local compensation as here shown masks the error contained in the assumption of perfect and local isostasy; in part because for many regions the ancient heterogeneities of mass hidden within the crust seem in reality to be greater than the heterogeneities of mass visible at the surface in topographic form and created by present gradational actions.

The results of this chapter converge with the lines of evidence previously considered and confirm them in showing considerable defects from isostasy for areas which are 100 km. or more in radius. This confirmation is to be expected, since it would be indeed remarkable if a crust, competent to carry such loads as the *geologic* evidence from erosion and sedimentation shows to be imposed, should give *geodetic* evidence of fairly local and nearly perfect adjustment between the topographic forms, developed by present external processes, and the variations in density imposed by past internal forces.

SECTION A

DEVELOPMENT OF CRITERIA FOR SPHEROIDAL MASSES

Separation of lithospheric from centrospheric outstanding masses.—Let the zone of compensation be regarded as the boundary of the lithosphere. At its bottom consider to exist a zone in which that lateral flowage takes place which is necessary for movements of isostatic readjustment and the maintenance through geologic time of a condition of more or less complete isostasy. Below it is the inner and more rigid core of the earth, the centrosphere. Let those excesses or defects of density above the zone of isostatic flow which are not in accord with the isostatic compensation of the topography be designated for convenience as lithospheric outstanding masses. Let all heterogeneities of density within any earth shell below the zone of isostatic flow be called centrospheric outstanding masses.

In his recent paper on the "Interpretation of Anomalies of Gravity,"[1] Gilbert calls attention to the fact that if abnormalities of density exist below the zone of compensation they will produce anomalies of gravity without these signifying real departures from isostasy. This is a very necessary addition to the theory of the cause of gravity anomalies and deflection residuals. As a test, Gilbert has calculated the influence of a right cylinder with vertical axis, of density ± 0.025, with height and radius each equal to 122 km., whose upper surface is at a depth of 122 km., thus reaching up to the bottom of the zone of compensation as given by Solution H. Such a cylinder would give a maximum anomaly of ± 0.023 dyne at the epicenter, a quantity of the same order of magnitude as the mean anomaly for the United States, 0.018 or 0.020 dyne.

In the application of this test to the earth it would appear, however, that two things should be noted. First, to account for the *mean* anomaly of 0.020 dyne the centrospheric masses would have to be several times as great as this cylinder, even for this depth of 122 km. to the top surface, since the maximum value of the anomaly occurs at the epicenter of the mass, and for a cylinder of

[1] *Professional Paper 85C, U.S. Geol. Survey,* 1913, pp. 35, 36.

the form postulated falls off rapidly with increasing horizontal distance from the epicenter. Second, the test mass has been taken as contiguous to the zone of compensation above and with that limited depth given by the hypothesis of uniform compensation. This gives it greater effect according to the law of inverse squares, but postulates either an indefinitely thin zone of isostatic flow at the bottom of the zone of compensation or a capacity in a thicker zone of weakness to maintain within itself heterogeneities of density similar to those of the lithosphere above and the centrosphere beneath. It is thought by the present writer that a more probable presumption is that the centrospheric heterogeneities which may exist are distinctly deeper than 122 km. and separated from the lithospheric outstanding masses by a thick zone which yields to broad inequalities of pressure either upon it or within it and therefore is incapable of maintaining notable inequalities of mass in this shell.

The reasons for this preliminary hypothesis are briefly as follows: The depth of compensation seems to be variable and to extend in some regions to as much as 300 km., even under the assumption of compensation uniformly distributed and complete at the bottom. Under a more natural assumption that isostatic compensation gradually disappears, those heterogeneities of density which give isostatic compensation would gradually diminish with depth and this diminution would extend to a considerably greater depth than 122 km. If heterogeneities which act isostatically gradually disappear, the heterogeneities which can be borne in excess should also be expected to diminish.

As to the nature of the shell immediately below the zone of compensation, Schweydar has recently analyzed mathematically the results of the measurements of earth tides by means of the horizontal pendulum.

The calculations were designed to test the presence or absence of a viscous zone between an elastic crust and elastic interior. It is concluded that even a magma bed with a viscosity as high as that of sealing wax at house temperatures and a thickness of but 100 km. cannot be present. The assumption in best agreement

with observations is that of the presence of a layer about 600 km. thick, slightly ductile (coefficient 10^{13} to 10^{14}), existing beneath an outer crust 120 km. thick.[1]

By postulating such a thick zone for isostatic flow, the viscous resistances are reduced and solid flow is made easier. It also is in conformity with the probability of a gradual change of physical state from the rigidity above into a less rigid and less stable tract. and this in turn into a more rigid interior. Now if such a thick viscous zone is incapable of supporting over broad areas loads imposed by abnormalities of density above, it should also be incapable of supporting such horizontal inequalities of mass within it, provided these are sufficiently large. But in order to produce the same gravitative surface effects as more superficial masses, the heterogeneities of this zone of viscous flow would in fact have to be much larger. A cylinder of the dimensions postulated by Gilbert, if of negative density and adjacent to another at the same depth but of positive departure from the mean density, would tend to be underthrust by the latter, and the denser would in turn tend to be overflowed by the lighter.

For these reasons it is not to be expected that the same departures from those densities giving isostatic equilibrium which could exist in a rigid shell above would extend immediately below.

Influence of centrospheric heterogeneity.—To test the question of the influence of heterogeneity below the zone of compensation a sphere will be considered. First, one whose center is at a depth of 319 km., 0.05 of the radius of the earth. As a second test, the influence will be determined of a sphere whose center is at a depth of 637 km., 0.10 of the earth's radius. For considering the attraction of a mass at points on the surface other than at the epicenter it is more convenient to take the mass as having the form of a sphere rather than a cylinder, since the mass of the sphere acts in all directions as if concentrated at its center. This favors, furthermore, the accentuation of the effects upon the surface over what they would be if the outstanding mass had a stratiform extension.

[1] Dr. Wilhelm Schweydar, "Untersuchungen über die Gezeiten der festen Erde und die hypothetische Magmaschicht," *Veröffentlichung des k.k. Preusz. geodät. Institutes,* Neue Folge No. 54, Leipzig, 1912 (B. G. Teubner).

The sphere having the same volume as a cylinder 122 km. in radius and 122 km. in depth will have a radius of 111 km. Let a radius of 100 km. and a density of ± 0.025 be assumed as the dimensions and mass of a standard sphere in this deep zone. If the center of such a sphere is at a depth of 183 km., the same depth as the center of Gilbert's postulated cylinder, the anomaly at the epicenter will be 0.021 dyne, whereas the cylinder gave an anomaly of 0.023 dyne. They are therefore nearly equal in effect. If the center of the sphere is placed at a depth of 319 km., making the top at 219 km., the anomaly at the epicenter becomes 0.0068 dyne. Consequently the variation in density or volume of the sphere would have to become three times as great in order that its maximum anomaly should equal the mean observed anomaly. But as the average anomaly is not measured at the epicenter, and the maximum anomalies, occurring at the epicenters, are several times the observed mean anomalies, this figure would have to be still further multiplied. To account, therefore, for the magnitude of surface anomalies, the disturbing spheres, if with centers at a depth of 319 km. and if of 100 km. radius, would have to have abnormalities of densities ranging up to 0.25 in order of magnitude. If the centers of the spheres were at twice this depth the abnormalities, to produce the same effect, would have to be four times as great in mass. In a region of which there is no precise knowledge such variations of density might well occur. The problem must therefore be investigated by means of the gradients which would result in the gravity anomalies and deflection residuals and a comparison of these with the gradients actually observed and plotted.

Distribution of surface forces for centrospheric spheres.—For masses as deep as these the curvature of the earth becomes of importance, but the complications which it introduces into the analytic treatment have been avoided by means of a graphic solution.

In Fig. 8A, the anomaly is calculated for the epicenter. Then the gravitative force at any other point on the surface, such as that having a dip angle θ, can be determined by squaring the inverse ratio of distance. Multiplying the force at the epicenter by this

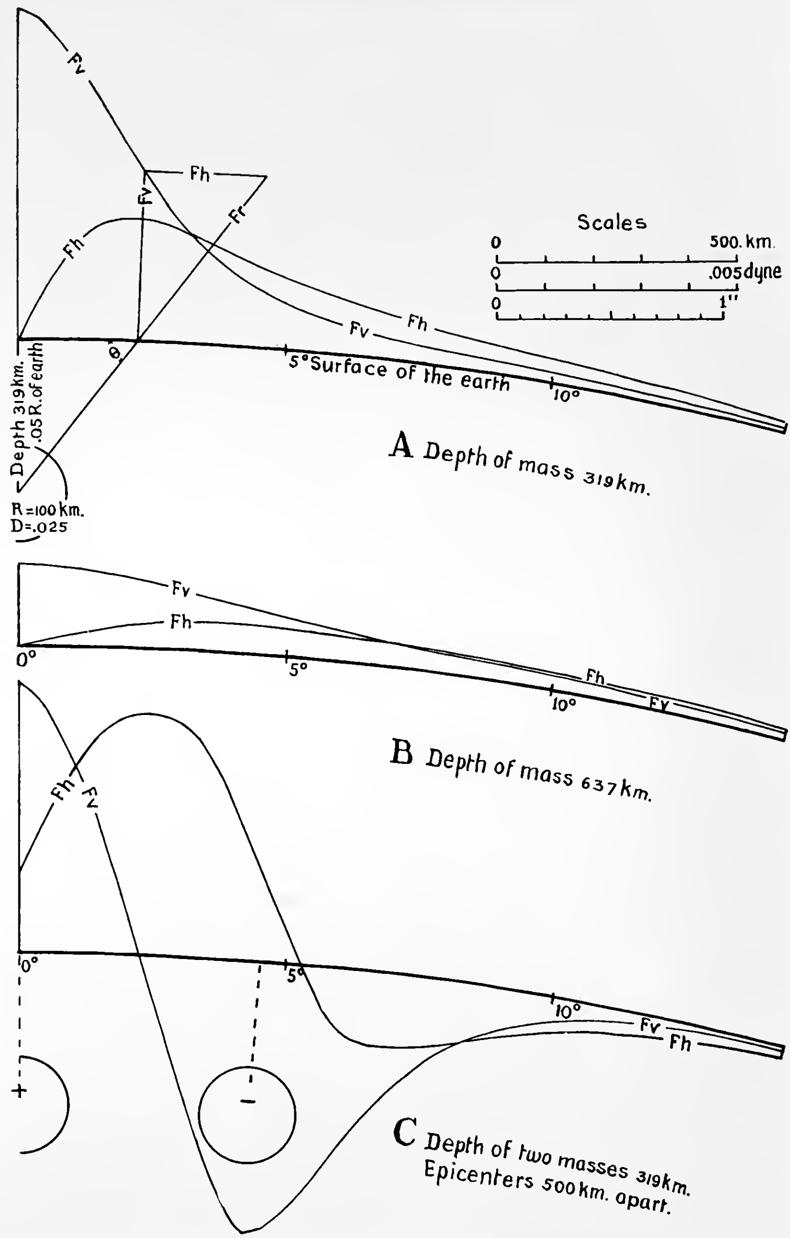

FIG. 8.—Curves showing values for the gravity anomalies *Fv*, and deflection residuals *Fh*, on the surface of the earth, for spherical masses situated at depths of 319 and 637 km., respectively. Ordinates measured at right angles to earth's surface.

factor gives the force at the second point acting in the direction of the radius of the attracting sphere. This value is laid off as *Fr* and then resolved into two components *Fv* and *Fh*, vertical and parallel respectively to the surface of the earth. The ratio of *Fv* to *Fh* is tan θ. With increasing distance from the epicenter θ becomes increasingly greater than it would be if the earth's surface were regarded as a plane. Therefore for distances of 5 to 10 degrees and more from the epicenter *Fv* begins to hold an appreciably higher ratio over *Fh* than it would if curvature were neglected. It is seen that $Fv=Fh$ for $\theta=45°$. Nearer the epicenter *Fv* is in excess; at greater distances *Fh* is the greater. *Fv* is a maximum for $\theta=90°$. *Fh* is a maximum for $\theta=55°$ if curvature be neglected. For the earth's curvature and a depth of 319 km. to the center of mass, θ is a maximum for $53° \pm$. The point giving this is at a distance from the epicenter of 0.75 the depth. The ratio of maximum *Fv* divided by maximum *Fh* is approximately 2.7. In Fig. 8C are shown the effects of two spheres of opposite sign but of equal mass. If these two spheres were superposed they would of course completely neutralize each other. Upon moving them horizontally apart to 1.5 times the depth, the maximum value of *Fh* becomes twice the value for a single sphere. This occurs halfway between them, and the value of *Fv* for this point is zero. The ratio of maximum *Fv* over maximum *Fh* becomes 1.1. Two equal masses of like sign would, on the contrary, give a maximum value of *Fv* and a zero value of *Fh* at a point halfway between them. These represent the extreme departures from the case of a single spherical disturbing mass. More distant masses show less overlapping of their fields of force and tend to have their individual effect upon a point between them neutralized by the larger number of masses acting from various directions. The values of *Fv* are much more under the control of the individual masses than are the values of *Fh*.

Returning to the single dominating mass of spherical form as shown in Fig. 8A, let the values of *Fv* and *Fh* be represented by ordinates as shown in the figure; then the surface representing the gravity anomalies, *Fv*, would be a dome of double curvature, like a craterless volcano; the surface representing the deflection

force, Fh, would be in the form of a caldera or crater ring. According as the attracting mass departed in its nature from a sphere these shapes of the force surfaces would be modified, but the general character of volcanic cone and caldera would remain. Negative masses would have the forms reversed. If the disturbing masses are at distances apart which average several times their depth, then a relief map of the resultant forces would resemble a volcanic field with the volcanoes isolated from each other. If the centers are much closer, the relief map would come more to resemble those lunar craters which show all degrees of superposition upon older craters. The deeper the masses the broader the volcano-like curves of forces upon the surface, but the lower will be the relief, unless the disturbing spheres increase in mass with the square of the depth. Stratiform-like masses, such as oblate spheroids or cylindrical disks, will show less pronounced effects than the equivalent spheres and will simulate somewhat the effects of spheres at a greater depth. The attempt to apply these principles as criteria to the published data must be deferred until the influence of abnormal masses in the zone of compensation has been considered and also in somewhat more detail the influence of masses in other forms than spheres.

Influence of spheres within the zone of compensation.—The forces produced by spheres within this zone are more readily treated analytically, since it will be seen that the curvature of the earth may be neglected. Otherwise this topic is to a considerable degree an extension of the last. The unit mass which it is convenient to adopt for this discussion is that of a sphere whose radius is 50 km. and density 0.100, one-half the mass of the sphere previously considered. Its center is taken at a depth of 64 km., 0.01 of the earth's radius, and approximately at the middle of the zone of compensation as given by Solution H. This gives the greatest abnormality of mass in the middle of the zone of compensation and will approximate to the mean effect of an outstanding density distributed uniformly throughout that zone. At the epicenter the attraction is wholly effective in producing gravity anomaly and is measured by the formula

$$F=\frac{dc(\frac{4}{3}\pi R^3)}{D^2}$$

In this d=density, c=constant of gravitation, R=radius, D=depth. Solving this equation for the values chosen gives

$$F = .0853 \text{ dyne}$$

Take the earth's surface as a plane and any point on it as located by the dip angle θ, made by a line from the point to the center of the sphere of outstanding mass. Then the vertical component Fv and the horizontal component Fh are given by the following equations:

$$Fv \text{ (dynes)} = 0.0853 \sin^3 \theta$$
$$Fh \text{ (dynes)} = 0.0853 \sin^2 \theta \cos \theta$$

To convert Fh into seconds of arc divide by 0.00475 and

$$Fh \text{ (seconds)} = 17.94 \sin^2 \theta \cos \theta$$

The maximum value of Fh occurs for $\theta = 55°$ and is 0.0328 dyne or 6.9 seconds. The curves for Fv and Fh are shown in the unbroken lines of Figs. 9 and 10. They are seen to be quite close in character to the curves of Fig. 8. Changes in the mass or depth of the sphere will serve to change only the scales of forces and distances so that these curves may be adapted readily to apply to all spherical masses situated within the lithosphere.

Influence of sum of intersecting spheres approximately equivalent to spheroids.—The analysis of the gravitative forces which a sphere exerts upon points in an external plane serves as a starting-point for the consideration of the problem of the influence of those unit masses of excess or defect of density which exist in the crust. As a further step, any one mass may be considered as approximating in form either to some oblate or prolate spheroid or to some ellipsoid of three unequal axes. But the equations for the forces exerted by spheroids upon an external plane are complicated and laborious to solve. A sufficient approximation to the influence of a spheroid may be made, however, by employing several intersecting spheres which together give an approximation to the right quantity and distribution of mass. The influence of the composite mass is readily attained by summing up the curves given by the modifications for the several spheres.

In Fig. 9 the unbroken lines, as previously noted, are the curves of force due to the single unit sphere. The broken lines show the

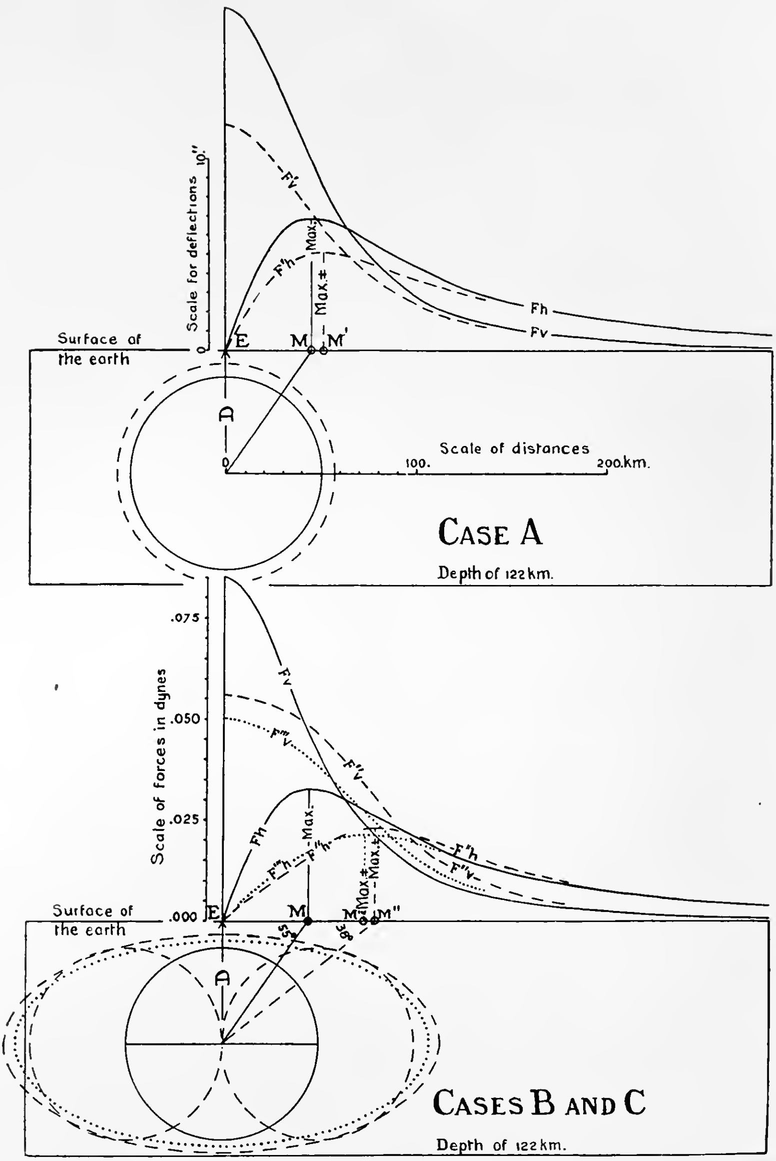

FIG. 9.—Various cases of horizontal and vertical components of the gravitative force due to a unit mass concentrated into a single sphere and expanded into several intersecting spheres. Cases A and B, three spheres in line. Case C, five spheres made by combining A and B and omitting one central sphere.

curves of force due to the same unit mass expanded into three spheres of the original dimensions and with centers 50 km. apart on a horizontal line. One-third of the mass is therefore in each sphere and the density of each is 0.033. In Case A the line joining the centers is at right angles to the vertical section plane. In Case B the line joining the centers lies in the section plane. The dotted lines show Case C. In this the unit mass is expanded into five spheres of the original size, each sphere possessing, therefore, one-fifth of a unit mass, and consequently a density of 0.020. In this case the five centers are arranged in a horizontal plane, the four outer spheres having their centers 50 km. from the center of the inner sphere.

The single sphere has a volume given by the formula $V=\frac{4}{3}\pi R^3$, in which $R=50$ km., and a density of 0.100. The three spheres have a volume therefore of $4\pi R^3$ and a density of 0.333. The three spheres make a solid of revolution whose semipolar axis is equal to $2R$, equatorial radius equal to R. Upon comparing this aggregate to a spheroid it is seen that the double density 0.667 of the intersecting portions compensates roughly for the two re-entrant zones on each side of the equator. To what regular spheroid does it approximate in its proportions? Let E be the equatorial radius of the spheroid and $2E$ the semipolar axis. The volume will be $\frac{8}{3}\pi E^3$, equal to the three spheres whose volume is $4\pi R^3$, or a single sphere of radius $1.44R$. Solving gives $E=1.14R$, $2E=2.28R$. The spheroid with these semiaxes is shown in broken lines in Fig. 9. This, then, is a spheroid which, if the density be taken as 0.033, is of exactly the same mass as the original unit sphere, or the three intersecting spheres, and which approximates in distribution of mass and in gravitative effect to these three spheres as shown in Fig. 9. The nature of the differences will be discussed later.

Case C shows five spheres of unit volume and of density 0.020 whose intersecting portions would consequently have densities of 0.040 and 0.060. In comparing the compound mass to an oblate spheroid these intersecting portions compensate roughly for the re-entrants between the spheres. The limiting dimensions of the

whole in the directions of the three principal axes are R and $2R$. Let it be required to find the value of the equatorial radius $2P$ and semipolar axis P of the oblate spheroid of equal volume in which the axes have these proportions. Then

$$\frac{16}{3}\pi P^3 = \frac{20}{3}\pi R^3$$

$$P = 1.08R$$

It is seen that the five spheres of density 0.020 have the same mass as a sphere whose radius is $1.71R$ and density 0.020; the same mass also as the unit sphere of density 0.100 and radius R and the oblate spheroid of density 0.020 and semipolar axis $1.08R$, equatorial radius $2.16R$. The vertical section of this spheriod is shown in dotted outline in Fig. 9. The distribution of mass and gravitative effect of the five spheres will be nearly the same as for such a spheroid. The nature of the differences, as in cases A and B, will be discussed later.

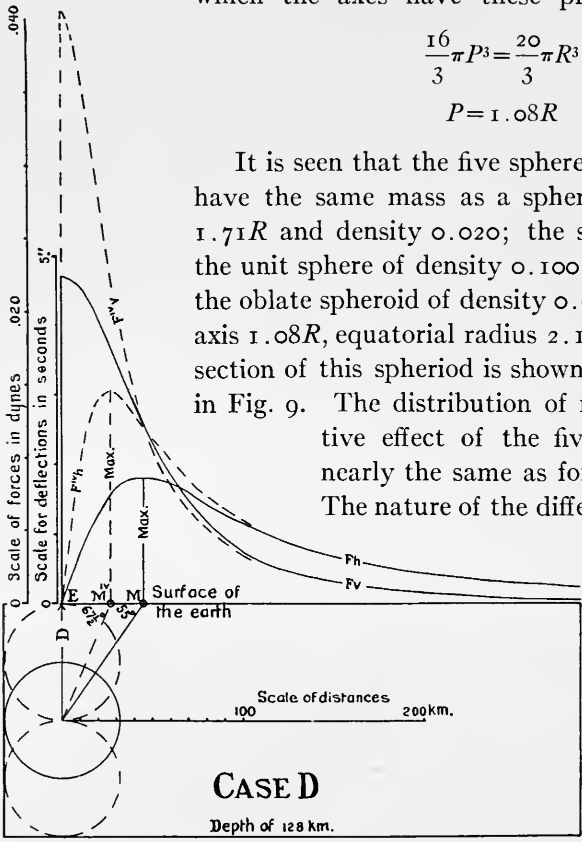

Fig. 10.—Horizontal and vertical components of the gravitative force due, first, to a sphere of 32 km. radius, density 0.100, depth to center 64 km., and second, to the same mass expanded into three such spheres with centers on a vertical line at depths of 32, 64, and 96 km.

The effect of the distribution of mass along a horizontal line and in a plane has been considered in cases A, B, C. There remains to be considered the effect of the distribution along a vertical line and in a vertical plane. Case D, Fig. 10, is given to show the effect of the distribution along a vertical line. The unbroken-line curves show the values of Fv and Fh for a sphere with density and depth corresponding to the unit sphere previously

used, but with radius of 32 km. instead of 50 km. The mass is consequently but 26 per cent of that of the unit sphere. Let this be expanded into three intersecting spheres with centers 32 km. apart and arranged in a vertical line. The composite mass will then extend from the surface to a depth of 128 km. and correspond closely in effect to a prolate spheroid with vertical axis.

The resulting values of the components of the gravitative force for these four cases which are of importance for the development of criteria may be tabulated as follows:

TABLE XXV

	Unit Mass		Ratio of Maximum Fv Divided by Maximum Fh	Value of θ for Maximum Fh
	Maximum Fv	Maximum Fh		
Sphere	0.085	0.033	2.6	55°
Case A	0.056	0.024	2.3	51±
Case B	0.056	0.023	2.4	38±
Case C	0.050	0.022	2.4	41±
	0.26 Unit Mass			
	Maximum Fv	Maximum Fh		
Sphere	0.023	0.009	2.6	55
Case D	0.041	0.015	2.8	67.5

To complete the series the curves should be drawn for five intersecting spheres arranged in a vertical plane analogous to Case V, first parallel and then at right angles to the plane of the section, making cases E and F, but the general character of the resulting curves may be inferred from the cases already given. Therefore, in order to abbreviate the discussion, an additional figure for cases E and F has been omitted.

It is seen from inspection of Figs. 9 and 10 and Table XXV that even for a constant mass and center of gravity the values of Fv and Fh change rapidly with the changing form of the mass. Upon the linear extension of a sphere into a form such as cases A and B the maximum value of Fv falls to two-thirds of its original value. For Case C it is still less. The ratio of the maximum value of Fv divided by the maximum value of Fh is also seen to change, but

more slowly, decreasing for the spreading-out of the mass in a horizontal direction, increasing for a linear vertical extension. For the spheroids whose axes are in the relation of 1 to 2 and to which the spheres are equivalent in volume, the changes would be still more marked. This is because the duplications of mass due to the intersecting spheres are near the center and in Case C a top view would show considerable deficiencies of mass in the equatorial zone between the spheres and the spheroid. These intersecting spheres are consequently more effective in producing gravity anomaly and somewhat more effective in producing deflection of the vertical in the zone of maximum deflection. For cylindrical disks of the same mass and proportions as the spheroids the changes away from the values for a sphere would be still greater, since a greater proportion of the mass would be removed from the center to the edges of the body.

Distinctive effects of individual spheres and spheroids.—In the subjection of deflection measurements to the hypothesis of isostasy Hayford found it necessary to consider the effects of topography to a distance of 4,125 km. and in the determination of gravity anomalies the topography of the whole earth and its compensation were considered. To what extent then do distant masses affect the local residuals and vitiate any attempt to analyze the effects of local masses? In answer, it is seen that the effects of distant masses are negligible. It is the great topographic contrast of ocean basins and continental platforms, to a lesser extent the large variations of relief within these areas, which require their effects to be considered to such a great distance. But it is found that this larger relief of the crust is nine-tenths compensated and the outstanding masses so far as known do not show any marked segregation as to sign within the continental areas as opposed to the oceanic areas. Furthermore, the unit areas departing markedly from isostatic equilibrium are much smaller than these major segments of the crust. Therefore where it is the effect of the outstanding masses which is under consideration they are seen to be individually small in comparison with the greater relief features of the globe, and further, they mutually cancel their effects. The local outstanding masses are furthermore of the same order of

magnitude as the more distant ones and therefore the effects of the distant masses sink to negligible quantities in comparison, in accordance with the law of inverse squares. The general agreement in the magnitude of the departures from isostasy, as shown by deflection residuals and gravity anomalies, shows, furthermore, that deep nucleal heterogeneities can have no large and broad regional effects, since such would affect the gravity measurements within a broad central circle to a greater degree than they would the deflections of the vertical. Therefore each region is seen to offer its local problems and the dominating centers of outstanding masses may be readily determined save where several such masses are contiguous, especially if of opposite sign. Let attention be given then to those features in the influence of outstanding masses which are indicative of the form and depth of the attracting mass.

Let the depth to the center of mass be D. Then it is seen that for ellipsoids near the surface in which the major axes are twice the minor axes the influence of form has mostly disappeared at horizontal distances from the epicenter or from $2D$ to $3D$, and at greater distances the curves become practically coincident. For greater departures from the spherical form the distances before the curves approach those given by a spherical mass are still greater. At these distances where the curves approach those of spheres, the effects of the form of the mass could not be distinguished, but the curves are so flat that neither could the effects of depth be readily evaluated. For instance, the curves of force at $4D$ to $6D$ for a sphere of mass M at depth D would be approximately the same as for a sphere of mass $4M$ at a depth of $2D$. Furthermore, at these distances from the epicenter the forces are so small in proportion to the maxima for the same mass that other outstanding masses would greatly change their value and prevent a correct analysis. Therefore, to be determinative, observations must be made at a number of points between the epicenter E and a distance not more than double that which at the point M gives the maximum value to Fh. It is seen that if a mass symmetrical about a vertical axis departs widely from the spherical form, this will be detected by noting the ratio of maximum Fv to maximum Fh, the latter being measured along any line radiating from the epicenter. If

the mass is unsymmetrical about a vertical axis, observations must be made along at least two lines at right angles to each other.

A minimum number of observations will define an isolated outstanding mass, but if several masses have their fields of force notably overlapping, a larger number of observations becomes necessary in order to differentiate their effects. An inspection of Figs. 9 and 10 shows that the shape of the curve of Fv between the epicenter and a distance where it falls to one-tenth the maximum value has more distinctive relation to the form of the immediately adjacent mass than has the curve for Fh. But the available geodetic data supply less information regarding the gradients of the gravity anomalies than for the deflection residuals. The latter are given along a certain belt of triangulation stations, whereas the gravity stations are located at long distances apart. Furthermore, but few of the gravity stations coincide with deflection stations. The present analysis will therefore rest upon the data giving the curve for Fh. This curve is flat at the top, so that the data will readily give the approximate value of the maximum but will not determine closely its distance from the epicenter. The value of θ will, however, be determined ordinarily on two sides of the epicenter by means of the deflection residuals and the mean will give a more reliable figure than either alone. But according to the form of the mass within those limits shown in Figs. 9 and 10 the value of θ may range from $38°$ to $67\frac{1}{2}°$. If the abnormal mass is assumed to have a spherical form, its center will lie at an angle of $55°$ below the maximum value of Fh and at a depth 1.4 the distance to the epicenter. The error in locating the points of epicenter and maximum Fh may cause the estimate of depth to be in error 20 per cent and yet this figure will show definitely whether the sphere lies within the zone of compensation or in the centrosphere. If the mass, however, is in reality a horizontally elongate mass, the change in the distance EM from the epicenter to the point of maximum Fh in two directions at right angles to the epicenter will show that fact. A check on the form of the mass may be obtained if the value of the deflection curve is known with fair accuracy to a distance from the epicenter of three times the distance of the maximum. Let the distance to the point of maximum value

be M. The ratios of the value of Fh at M to the value at $3M$ are given below as measured from the curves.

RATIOS OF Fh AT M TO Fh AT $3M$

Sphere	2.4
Case A	2.1
Case B	4.3
Case C	3.4
Case D	2.0

It is seen that, from the difficulty in the precise location of M and hence the difficulty in locating a point as at $2M$ or $3M$, and furthermore the probability that other masses may influence to a degree not readily determinable the value at $3M$, this test cannot ordinarily be determinative. However, for spheroids with polar axes vertical, markedly prolate masses give a ratio distinctly smaller than for a sphere, the curve of deflections falling off more abruptly; markedly oblate masses, on the other hand, give a ratio distinctly larger than that for a sphere, the curve of deflection beyond the point of maximum being flatter.

If the outstanding mass approximates to an oblate spheroid with polar axis vertical, then the assumption of a spherical nature will locate the center of mass too deep and imply a greater mass than really exists. If, on the contrary, the form of the outstanding mass approaches the form of a vertical prolate spheroid, the interpretation of the deflections as caused by a sphere would locate the center of figure too high and give it too small a mass.

Suppose the curves for Fh shown in cases C and D have their maxima well determined in position and in magnitude, but that the values of the curves at distances two or three times beyond are not accurately known. Let these maxima be interpreted as produced by spheres. The depth and masses of the spheres which give these maximum deflections will be too great by the amounts shown in the following tabulation (Table XXVI, p. 462).

Where the data are sufficiently complete the form and depth of mass may both be determined, though a high precision is not to be expected. But in most cases with the present geodetic data the form of the mass will not be determinable and all that can be

done is to interpret the deflections as produced by spherical masses. What then are the geological suggestions as to whether vertical prolate or oblate forms may be expected to characterize the larger outstanding masses? In the one case the error of interpretation will be to make the masses appear too small and shallow; in the other case, to make them appear too great and deep.

TABLE XXVI

ERRORS DUE TO INTERPRETATION AS SPHERES OF UNIT MASSES AT DEPTH *D*

Form	Data		Assumption	Resulting Interpretation	
	Fh Max.	*EM.*	θ	Depth to Center	Mass
Case C (True)	4″.5	74 km.	41°	1.0*D*	1.0*M*
Interpretation as a sphere	4.5	74 km.	55	1.6*D*	1.8*M*
Case D (True)	3.05	27 km.	67.5	1.0*D*	1.0*M*
Interpretation as a sphere	3.05	27 km.	55	0.6*D*	0.6*M*

Stocks, and especially volcanic pipes, approach in form to vertical cylinders, but these are merely connecting structures. On the other hand, mountain ranges and geosynclines, although linearly extended, are of breadth which is great in comparison with the depth of excess or defect of density. Laccoliths and regional extrusions are also broad in comparison with depth. The relations as regards the great intrusive masses are not so clear, but erosion exposes batholiths over progressively greater areas; and whole provinces which exhibit regional metamorphism give suggestions that they are underlain by widespread igneous bodies. The hydrostatics of the magmas and their differentiation into masses of unlike density would also give tendencies to layers and horizontal extensions of the larger masses of abnormal density. These would depart, then, from the form of spherical masses in the direction of oblate spheroids with their equators in a horizontal plane. Narrower belts of disturbance like that which passes through Washington, D.C., may, on the other hand, tend to have the form of vertical plates. Therefore in none but the smaller and connecting structures are there geological suggestions of vertical prolate form.

The summation of this discussion shows that if in the first assumption as to the form of abnormal masses they be taken as spheres, then the determination of the depth of the center of mass by means of the curve for Fh and the location of the point of maximum value is more likely to overestimate than underestimate the depths and masses. As the object of this investigation is especially to find the depth of masses and to test the hypothesis of centrospheric heterogeneity as a cause of deflection residuals and gravity anomalies, it is desirable to have the error of interpretation in the direction of indicating a depth too great rather than too small. Therefore the initial assumption that the outstanding masses are spheres is justified by the geologic probabilities and is found in Section B to be justified by the geodetic evidence. The next topic will therefore develop further the subject of the interpretation as spheres with the view to utilizing the geodetic data.

Depths of spheres whose epicenters are not on the line of traverse.—If the primary purpose of a geodetic investigation were the determination of the location of the epicenters of abnormal masses and then the measurement of their form, size, and depth, a series of gravity and deflection measurements could be made in a line passing above the mass and near the epicenter. The preceding discussion would then directly apply. In only a few localities, however, will a line of triangulation stations, located in connection with the measurement of the earth's surface, pass approximately over the center of a large outstanding mass. How then, from the locations and values of the deflection force along any linear belt of measurements, shall the location of the epicenter and depth to the center of an abnormal mass to one side of the line of traverse be determined?

In Fig. 11 is developed a method for the solution of this problem. In accordance with the preceding discussion and the reconnaissance nature of a first investigation, let it be assumed that isolated abnormal masses approach a spherical form; that is, that a mass may be regarded as concentrated at a point. Take the center of the mass as the center of co-ordinates and the axis X–X as lying parallel to the line of traverse. The epicenter is at E. Then the vertical distance from the center to the epicenter is D. The

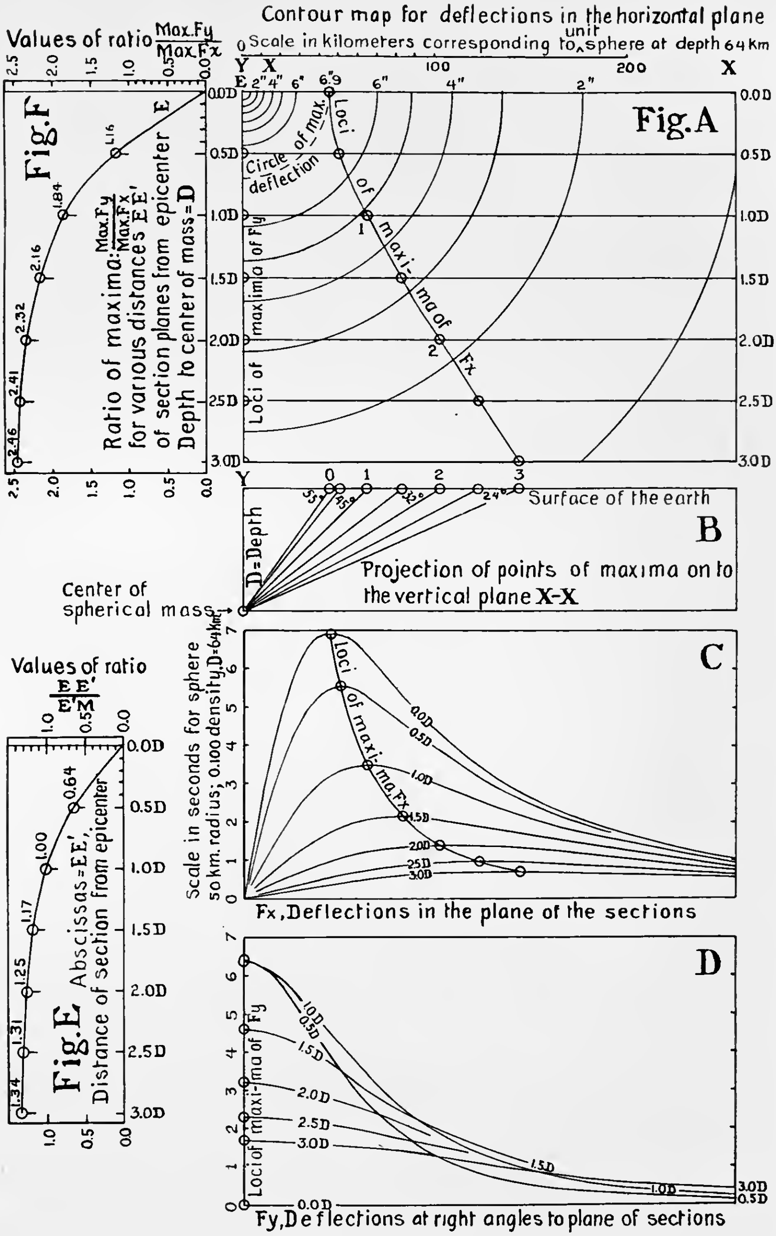

FIG. 11.—Values of the components of Fh on traverse lines which pass at various distances EE' from the epicenter E of a spherical mass at depth D; EE' being measured in terms of D.

distance EE' from epicenter to the traverse line is y. Then the co-ordinates of any point on the surface are x, y, and D. At any such point the component in the horizontal plane of the gravitative force due to the outstanding mass is Fh. But Fh is directed radially from all points to the epicenter. Fh, consequently, is subdivided into two components at right angles. Let that acting in the line of the section be called Fx, that at right angles be called Fy, parallel respectively to the X and Y axes.

The value of Fx for any point is given by the equation

$$Fx = dc\frac{4}{3}\pi R^3 \frac{x}{(x^2+y^2+D^2)^{\frac{3}{2}}}$$

and the value of Fy is given by the equation

$$Fy = dc\frac{4}{3}\pi R^3 \frac{y}{(x^2+y^2+D^2)^{\frac{3}{2}}}$$

Fig. 11 shows the results of the solution of these equations. In Fig. 11A is drawn a contour map of the deflection force produced by the unit sphere. The deflection force Fh at any point is measured by the contour map and is directed toward the epicenter. The lines of equal deflection are seen to be circles with center at E. They show in plan the values which were shown in section by the full lines for Fh in Figs. 9 and 10. The contours, as previously discussed, are seen to give the form of a volcano whose crater has a rounded rim and a conical interior reaching to the epicenter.

Now let a number of parallel sections be taken at horizontal distances from the epicenter equal to $0.0D$, $0.5D$, etc. The curves for Fx for each section are shown in Fig. 11C and for Fy in Fig. 11D. The points of maximum value for each section are indicated in A, B, C, and D and through these points are drawn the curves which are loci of maxima. For Fy there is a single maximum for each section and this is situated at $x=0$. For Fx there are in each section two equal maxima but of opposite sign, one for a plus, the other for an equal minus value of x. In Fig. 11C only one side of the curve is shown, that for plus values of x.

Fig. 11B shows the points giving maximum values of Fx projected onto the vertical plane passing through X–X. The dip

angle θ measures the slope from the point of maximum value to the center of mass. Here is shown its value when projected onto the plane X–X. This projected angle is seen to grow smaller with each more eccentric position of the section plane.

For section $0.0D$ the real value of θ is 55°
For section $1.0D$ the projection of θ' is 45°
For section $2.0D$ the projection of θ'' is 32°
For section $3.0D$ the projection of θ''' is 24°

Therefore it is seen that if the traverse line were assumed to pass through the epicenter of all disturbing masses, the error introduced would be to show the center of mass deeper than it really is. The nature, however, of the geodetic data permits this assumption to be eliminated and the distance EE' to the section plane to be approximately determined. An error up to $0.5D$ will not involve much error in the resultant depth as determined by the projection of θ. At each station along a line of triangulation[1] both Fx and Fy are determined and their resultant points toward the center of gravitative control. Each station gives an independent determination of this resultant and the intersection of two resultants if accurately determined and due to the gravitative force of a single symmetrical mass would give an accurate location of the epicenter, measuring its distance and direction from the traverse line. The data in many cases permit as many as three or four resultants to be drawn, the size of the triangle of their mutual intersections showing to what degree the forces may be ascribed to a single center. The relative positions of the line of section and epicenter of mass are thus in many cases approximately established.

But although the relative position of epicenters and traverse line are thus ascertained, the depth of the masses remains to be solved. In Fig. 11 the distance of the traverse line from the epicenter is given in terms of D, but this is the unknown. Two independent methods lead up to the solution of D.

First, on any line of section occurs a zero point for Fx. Let this be called E'. On each side of the zero point for Fx occurs a

[1] As shown in illustration No. 3, Hayford, *Supplementary Paper.*

maximum value for Fx. Let the point of this maximum be called M. These two values are given by the geodetic data. Then for any spherical mass the ratio of $\frac{EE'}{E'M}$ increases with increase in the ratio of $\frac{EE'}{D}$ but not as a rectilinear function. This relation of ratios is shown graphically in Fig. 11E, in which the abscissas are the values of $\frac{EE'}{D}$ and the ordinates are the corresponding ratios of $\frac{EE'}{E'M}$. This ratio may be determined from the geodetic data but *from the location of the maxima, not their amount.*

The second method for determining the depth depends upon the ratio of the maximum value of Fy for any traverse line to the maximum value of Fx for the same traverse line, thus being dependent upon the *relative values of the maxima and not their location.* This ratio also increases with increase in the ratio of $\frac{EE'}{D}$ but not as a rectilinear function. The maximum Fx for a section at any distance from E is shown in Fig. 11C. For example, if $E'E=0.5D$ the maximum Fx for the unit sphere is $5.52''$. The maximum Fy is shown in Fig. 11D, and for $E'E=0.5D$ is $6.42''$. The ratio of 5.52 to 6.42 is 1.16. These ratios are shown in Fig. 11F for all traverse lines up to a distance of $3.0D$ from the epicenter. The value of this ratio is given by the geodetic data for any traverse line and hence the distance to the epicenter is given in terms of D.

In conclusion on this topic it may be said that the curves shown in Figs. 11E and 11F are independent of the mass or volume of the sphere, depending only upon its depth, and are adapted to use with the geodetic data. It is seen from both curves that the significant ratios change in value rapidly with increasing distance of the traverse line from epicenter up to a distance $E'E=D$, but beyond this point the change in the value of the ratios becomes progressively small as compared to a change in the distance of the section plane. The method is therefore well adapted for

determining the depth of the outstanding masses assumed as spheres, provided the section line is not farther from the epicenter than the latter is above the center of mass. The method may be used, however, with less precision for distances of the traverse line $E'E$ up to $2D$. Beyond this distance, however, influences of other masses or errors in the geodetic data would be likely to give wholly erroneous results, not distinguishing between a large excess of mass at a great depth or a smaller one at a much less depth.

[*To be continued*]

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THE STRENGTH OF THE EARTH'S CRUST

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PART V. THE DEPTH OF MASSES PRODUCING GRAVITY ANOMALIES AND DEFLECTION RESIDUALS

SECTION B

APPLICATIONS OF CRITERIA TO DETERMINE THE LIMITS OF DEPTH, FORM, AND MASS

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DEPTHS INDICATED BY THE MAP OF DEFLECTION RESIDUALS

General relations shown by deflections.—Hayford gives a plate[1] which shows all the residuals of Solution H. These are laid off as arrows and show graphically the magnitude and direction of the portions of the deflections which are outstanding after allowance is made for the deflections calculated according to Solution H. They therefore show the excesses or deficiencies of mass in the crust

[1] *Supplementary Paper*, illustration No. 3; also Bowie, 1912, illustration No. 5.

as measured against the demands of the hypotheses made in that solution. In Fig. 12A is reproduced a portion of Hayford's chart. A general inspection of the map of the residuals of Solution H shows that many of the large deflections of opposite sign lie comparatively close together. On a line connecting two stations, Fx is the component of the deflection which lies in that line, Fy is the component at right angles to that line. In most cases not enough stations are located on an approximately straight line to permit well-defined curves to be drawn for Fx and Fy. But it has been shown that for spheres and other concentrated masses the curve for Fx rises steeply from zero to maximum value and sinks away more gently beyond. Even for flat disks the outer part of the deflection curve will be flatter than for the inner part. Random locations on the curve are therefore more likely to give the maximum measurement at some point beyond the real maximum rather than at some point between the epicenter and the real maximum. Using these stations giving maxima for Fx as if they were at the points of real maxima will therefore give on the average too great a distance from the center to the point of real maximum Fx and consequently too great a depth to the centers of attraction. Interpreting the disturbing masses as spheres is also an assumption likely to give too great a depth, as is indicated later. Minor centers of outstanding mass will affect the positions of the points of maximum value, but in a sufficient number of examples this effect will largely cancel out. The tabulation of the distances measured from Hayford's map between ten pairs of notable Fx maxima is given in Table XXVII.

It is seen that the distance between these maxima is more largely dependent upon the length of the sides of the geodetic triangles than upon the depth to center of mass, since in less than half of these illustrations did a station fall between the two maxima. The distance between the real maxima is then probably somewhat greater than 86 km., the average of the six distances without intervening stations, but is probably somewhat under the general mean of 110 km. This mean distance of 110 km. between ten notable maxima of Fx corresponds to a mean depth of spheres of 79 km. Considering the various assumptions made, it is seen that

the mean depth of masses producing these deflections is probably much less than 79 km. They belong, therefore, to the outer half of the zone of compensation.

TABLE XXVII

DISTANCES BETWEEN ADJACENT LARGE DEFLECTIONS OF OPPOSITE SIGN

Locality of Mass	Sign of Mass	Numbers of Stations	Stations between	Distances between in Km.
New Jersey	+	255–142	0	110
Kentucky–Ohio	+	85–84	0	100
Georgia–Florida	+	292–294	1	160
Florida	+	299–300	0	145
Michigan–Indiana	+	344–346	1	165
Illinois	+	75–74	0	90
Nebraska	+	327–329	1	170
Colorado	−	59–57	1	100
California	−	238–236	0	50
California	−	245–246	0	22
			Mean	110

There are other areas, however, as in the Adirondacks, Maine, Michigan, and the Great Basin, where the distance between the large deflections of opposite sign is considerably greater. So far as this relation goes they could be due either to broad outstanding masses in the zone of compensation or to much greater but more concentrated masses in the nucleus beneath. But the general relations to the magnitude and location of the gravity anomalies as discussed later under that subject suggest that in so far as the evidence is determinative these broader areas are also due to broad excesses or deficiencies in the outer crust, not to masses in the centrosphere. The data are not, however, in all areas of a sufficiently complete nature to give determinate solutions. In other areas, however, detailed study following the lines of criteria previously developed can bring out very definite results in regard to the location and depth of masses in spite of the interference of the fields of force from various centers. An example of what may be done by a detailed examination is shown under the next topic.

Detailed study of the Texas–Kansas region.—Fig. 12A shows the deflections as given on a north-south line of triangulation 1,000 km. in length. The gravity anomalies are shown for distances of 200 km. on each side of the traverse. The stations are sufficient in

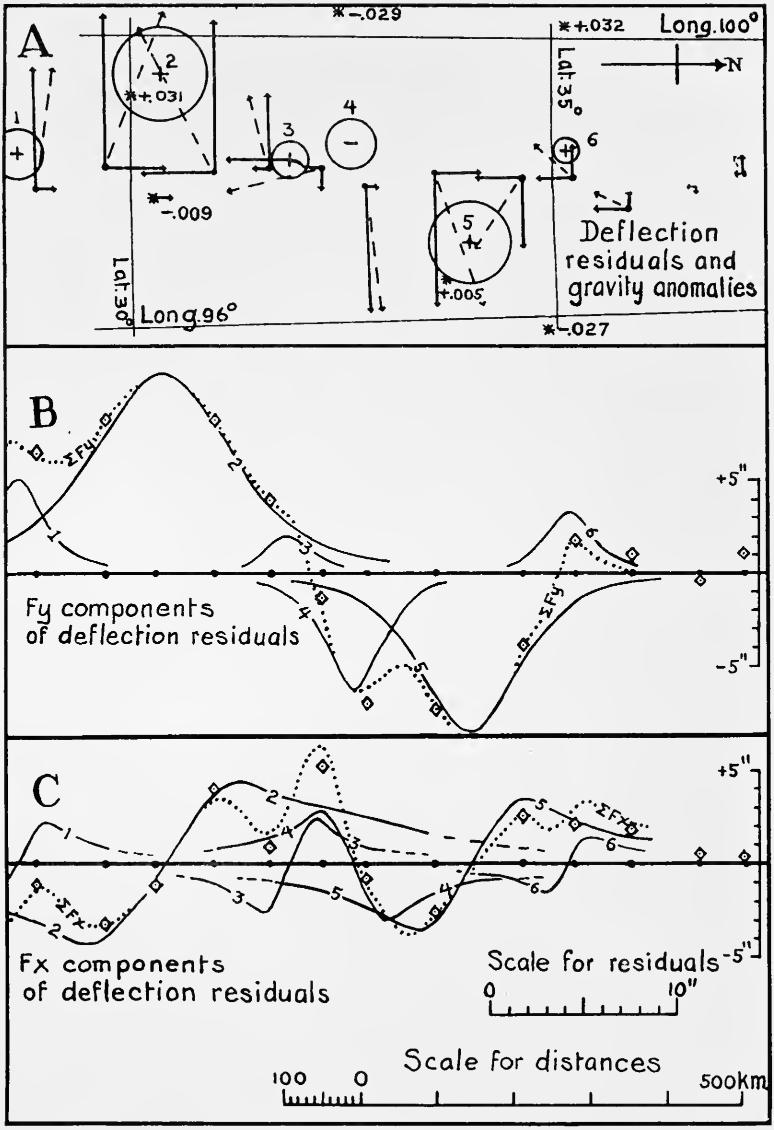

FIG. 12.—Residuals of Solution H and gravity anomalies in Texas, Oklahoma, and Kansas, with the interpretation of the outstanding masses in terms of equivalent spheres.

number and sufficiently close to a straight line to permit the application of the principles previously discussed.

The residuals are given in the north-south and east-west directions. The broken lines give the resultants. Their convergence indicates that there are two large and controlling positive masses marked on the map as 2 and 5. To account for the local variations shown by the resultants from station to station it is necessary, however, to locate smaller masses of positive or negative nature approximately as shown at 1, 3, 4, and 6. There must be of course many other centers of moderate disturbance within the area of 400,000 km. which is shown, but such as exist are far enough from the line of section not to exert an appreciable influence. It is noteworthy that gravity stations only 200 km. from the line of the traverse can show anomalies as large as -0.029, $+0.032$, and -0.027 dyne without the masses which give these anomalies showing appreciable control over the deflections on the line of traverse. Their areas of influence are therefore restricted. The limited influence of these masses giving anomalies somewhat above the average and at a moderate distance, and the small masses locally modifying the deflections both serve to show the importance of nearness of location. This limitation of control over the deflections, restricted to distances of less than 100 to 200 km., is itself an indication that these outstanding masses lie within the zone of compensation, otherwise their effects would be more far-reaching.

The residuals permit, however, a much more detailed solution to be made. As a first approximation assume the outstanding masses to be spheres. Figs. 12B and 12C. show the results. This is not merely an arbitrary adjustment of curves and one of a number which might be devised. On the contrary, it has been shown in the discussion of Section A and especially in Fig. 11 that the ratio of the two maxima of the deflection components, Fy and Fx, and the ratio of EE' to $E'M$ hold a definite relation to the distance and depth of the center of the sphere. Therefore if curve 2 be drawn in proper proportion and as shown in B in order to satisfy the demands of the y component, then the maximum value of Fx must not be over 40 per cent of the maximum value for Fy, even if the center

of mass is close to the surface. It may be any value less than 40 per cent of maximum *Fy*, according to the depth of the center. But having chosen that ratio which appears to fit the demands of the data, the distance of the point of maximum *Fx* from the point of zero value becomes also fixed. The curves numbered 2 in Figs. 12B and C must therefore satisfy between them the demands of the ratios shown in Figs. 11E and F. The value of EE' as deduced from either curve must be the same.

The sum of all the *Fy* curves in 12B is marked $\Sigma\ Fy$ and must pass through, or close to, the points which measure the values given by the deflection residuals. These points are shown as small rectangles in B and C and give ordinates which correspond with the size of the components of the residuals as shown in A. In drawing B and C the adjustment of the curves to give the proper values to $\Sigma\ Fy$ and $\Sigma\ Fx$ resulted in a slight readjustment of the centers of mass as shown in A. The positions as shown in Fig. 12A have been determined from the curves below, and their approximate agreement with the initial indications of the resultants is a check on the validity of the solution. It is seen that the epicenter of a mass should not lie on the exact intersection of any two resultants, since at the point of measurement several masses have an appreciable influence upon the direction of the resultant. The adjustment of the curves is therefore the best way of determining finally the best location of the epicenters.

The measurement of these curves gives the tabulation of data shown in Table XXVIII.

The depth to the centers of the equivalent spheres having been solved by means of the ratios given in Figs. 11E and 11F, the masses of these spheres are ascertained as follows. In Fig. 11 the value of the maximum deflections for *Fy* and *Fx* due to the unit sphere are shown for various distances of the section line from the epicenter. For example, for $EE'=1.5D$, max. *Fy* is $4.6''$. Now for sphere No. 4, Fig. 12A, $EE'=1.5D_4$ and the max. *Fy* is $6.2''$. But D for the unit sphere is 64 km. whereas D_4 is 31 km. Now the magnitude of the deflections for points similarly situated in two fields of gravitative force will vary directly as the respective masses and inversely with the squares of the distances. This may be put into

a formula as follows: Let there be two masses M and M_n with centers at depths D and D_n; below a horizontal plane. For points on the plane similarly situated with respect to their respective centers let the components of the deflection force be Fy and Fx for the one, Fy_n and Fx_n for the other. Then

$$M_n = \frac{Fy_n D_n^2}{Fy D^2} M; \quad \text{also } M_n = \frac{Fx_n D_n^2}{Fx D^2} M$$

The results of the application of this formula are shown in the last column of Table XXVIII. They have been carried out to two significant figures, but the second is not to be regarded as accurate,

TABLE XXVIII

INTERPRETATION OF RESIDUALS IN TERMS OF EQUIVALENT SPHERES

No. of Sphere	Data								Results	
	From Fig. 12			Fig. 11F	From Fig. 12			Fig. 11E		
	Max. Fy	Max. Fx	$\frac{\text{Max. } Fy}{\text{Max. } Fx}$	EE' in Terms of D	EE'	$E'M$	$\frac{EE'}{E'M}$	EE' in Terms of D	Depth in Km.	Mass in Terms of Unit Sphere Whose $R=50$ km. $d=0.1$.
1..........	5.1	2.3	2.20	$1.7D_1$	42	35	1.20	$1.7D_1$	25	$+0.20M$
2..........	10.8	4.4	2.46	$3.0D_2$	125	95	1.31	$2.5D_2$	40 to 50	$+2.5$ to $2.9M$
3..........	2.0	2.5	0.80	$0.3D_3$	13	33	0.40	$0.3D_3$	43	$+0.17M$
4..........	6.2	2.9	2.13	$1.5D_4$	47	40	1.17	$1.5D_4$	31	$-0.32M$
5..........	8.4	3.5	2.40	$2.5D_5$	88	67	1.31	$2.5D_5$	35	$+1.10M$
6..........	3.3	1.4	2.35	$2.2D_6$	34	27	1.27	$2.2D_6$	16	$+0.07M$

even if the original data are accurate to the second place. This is because the error of the square of a quantity is approximately twice as great as the original error, and for values EE' above $1.0D$ the error in even the first power of D is appreciably greater than the error in the measured quantities. This of course is a consideration of the error in the determination of the depth and mass of the hypothetical spheres, not a consideration of the errors in the deflections themselves, nor related to the fact that the masses are in reality not spheres.

It has been shown previously that the interpretation of the outstanding masses in terms of spheres will give depths too great unless the real masses have their greatest dimension vertical. For the same reasons the hypothetical spheres will be of greater mass

than the real horizontally extended bodies in order to produce the same deflections as those observed.

The question arises then as to the real form and mass of the bodies interpreted here as spheres. The supplemental data at hand yield evidence only in regard to No. 2. This, however, is the greatest of the six outstanding masses in this series. The supplemental data consist of observations on gravity anomalies at three localities whose distances from the epicenter of No. 2 are as follows:

RELATIONS OF GRAVITY ANOMALIES TO THE EPICENTER OF MASS No. 2

a,	47 km. S. E.	+0.031
b,	163 km. E.	−0.009
c,	250 km. N.N.W.	−0.029

Now the curves for *Fv* and also for the *Fy* component of *Fh* show with increasing distance from the epicenter a rapid fall from the maximum value. These effects of the gravitative force due to outstanding masses are consequently markedly local and the nature of the mass No. 2 must have a distribution such as to account for the anomaly of +0.031 at 47 km. from the epicenter. The interpretation must not give a form which will exert marked influence upon those points distant 163 and 250 km. The dominating influence of mass No. 2 on the value of *Fv* appears therefore to be confined to distances within 125 or 150 km. of the epicenter. But the two limiting spheres of mass 2.5M and 2.9M respectively give the relations shown in the first two lines of Table XXIX.

TABLE XXIX

INTERPRETATION OF MASS No. 2

Form	Mass in Terms of Unit Sphere	Depth to Center in Km.	Radius in Km.	Height in Km.	Excess Density Above Mean	*Fv* at Epicenter in Dynes	*Fv* at 47 Km. in Dynes
A. Sphere.....	2.5	40	40	80	0.5	0.542	0.148
B. Sphere.....	2.9	50	50	100	0.3	0.405	0.157
C. Cylinder....	2.5	40	200	10.4	0.1	0.035	....
D. Cylinder....	0.6	20	100	5.2	0.2	0.035	....

The interpretation of the deflections as due to spheres gives a gravity anomaly at the epicenter of the sphere between four and six times larger than the largest yet observed in the United States.

At the distance of 47 km. the anomaly would be from 0.148 to 0.157 dyne, five times the observed value of 0.031. Clearly then the initial interpretation as a sphere, although it satisfies the deflection residuals of the line of traverse, is far from the truth. The mass must have horizontal dimensions much greater than the vertical. Assume for trial that the mass has the form of a vertical cylinder with the same mass and depth to center as the sphere but of a proportion of height to breadth which shall satisfy approximately the gravity anomaly. The result is shown in the third line of Table XXIX. The gravity anomaly of 0.035 at the epicenter would correspond in a cylinder of these proportions to a value only slightly less at 47 km. But the radius of the cylinder, 200 km., is now far too great. Other cylinders of similar form will, however, give the same anomaly at the epicenter if the depth and dimensions are all divided by any number, n, and the density multiplied by the same number. This gives a series of similar cylinders in which the density varies inversely with the dimensions. Of such a series that shown in the fourth line, obtained by giving n a value of 2, comes fairly close to satisfying all the requirements. The exact degree of adjustment which would be needed to satisfy both the gravity anomaly at 47 km. and the deflections on the line of traverse has not been calculated. If this were done and the dimensions adjusted accordingly, it would complete a second approximation to the real form and mass of No. 2. Such an extended treatment of the subject would, however, be beyond the immediate purposes of this article and beyond the limits of space which it should occupy. The data also are at present hardly of a sort which would justify further computations. It should be emphasized, however, that such a complete investigation is not difficult and would require but little further data, properly chosen, to check the conclusions.

In the first approximation, the mass was assumed to have a uniform distribution about a center, giving a sphere. In the second approximation, the vertical axis is assumed to be different from the horizontal axes, but the latter being kept alike, the horizontal section would still be a circle. A single observation of the gravity anomaly near the epicenter suffices to give this second approximation. The third approximation would be to consider the three

co-ordinate axes of the mass unlike. This would require some observations in two directions at right angles from the epicenter. More complete data consisting of both deflection and gravity observations would of course give still closer approximations toward the real form and depth of the mass. What it is desired to show here, however, is that the interpretation of this large mass as a sphere gave a depth to the center of mass about twice too great and a mass perhaps four times too great. This result is in line with the general deduction previously made in regard to the direction of the error involved in the interpretation of deflection residuals as due to spheres. It contributes its individual testimony to show that the masses producing the notable gravity anomalies and deflection residuals are situated within the zone of isostatic compensation and more especially in the upper part of that zone.

In regard to the large positive mass No. 5 the data are less determinative. At a distance of 60 km. southeast from the epicenter the anomaly is only +0.005 dyne. At 150 km. northeast it is −0.027 dyne. It thus appears that there are some large negative masses easterly of No. 5. As this, however, is on the side away from the line of traverse the problem of the real form and mass of No. 5 is at present indeterminate.

The adjustment between the deflection curves due to spherical masses and the values of the deflection residuals has been made closer, perhaps, than the probable values of the residuals. Furthermore, the residuals of Solution G, if they had been given for this region, would have required a somewhat different distribution of masses. A solution for that depth of compensation which would reduce to the smallest quantity the sum of the least squares of the residuals of this area 1,000 km. long and 400 km. wide would be still somewhat different. That local solution which would give the smallest residuals would be such as would make small the algebraic sum of the positive and negative masses, but the difference in mass between positive and negative centers would not be much reduced. The depth to the centers of mass would be the quantity most affected by a change in the hypothesis regarding the depth of compensation. These epicenters then, in so far as the accidental errors do not vitiate the values of the residuals, are realities in nature.

In view of this analysis of the data given in Fig. 12 and in Table XXVIII, it is to be concluded that for this region even the larger outstanding masses from Solution H, capable of exerting a notable influence on the *Fx* component of the deflections to a distance of 400 km. or more, appear to have their centers not deeper than 20–25 km. Their mass is consequently within the outer half or even the other third of the zone of isostatic compensation as given by Solution H. There is further no evidence of centrospheric heterogeneity.

OTHER INDICATIONS REGARDING DEPTH OF OUTSTANDING MASSES

Deflections by linear or dike-like masses.—The resultants of the deflection residuals are shown by broken lines in Fig. 12. They show a tendency to converge toward centers. This is true in general for the whole United States, as shown in Hayford's illustration. This tendency to convergence indicates that the dominating outstanding masses may usually be regarded in a first or second approximation as symmetrical with respect to a vertical axis. In contrast, however, to this rule the residuals of Solution H in the vicinity of Washington[1] indicate an outstanding mass with a northeast-southwest extension of at least 120 km., whereas the breadth is probably not more than 20 km. This narrowness is shown by the limited distance between the large residuals of opposite sign in a northwest-southeast direction. The linear extension is shown by the parallel rather than radial arrangement of the resultants. The mass gives rise to large deflections for a distance of as great as 100 km. from the sides, but its influence dies out somewhat beyond.

It is clear from these relations that the assumption of a form of the mass symmetrical about a vertical axis for the purpose of determining the depth of the center would not be justifiable. Other assumptions which might be made in order to subject the mass to mathematical investigation would be to consider it as a horizontal prolate spheroid, or as a horizontal linear mass at a certain depth, cylindriform, or as a vertical plate. The latter would be preferable. For a quantitative solution of its dimensions and mass it would be desirable to have some observations farther to the northwest.

[1] Hayford, Supplementary Paper, illustration No. 4.

The following qualitative conclusions may, however, be drawn from an inspection of the residuals.

Maximum deflections are found on each side of the axial line not more than 40 km. distant. The deflections continue large for at least twice this distance but not for three times this distance. The existence of large residuals so close to the axial line shows conclusively that the outstanding mass is within the zone of compensation and apparently within its outer half, but the maintenance of the size of the deflections without much change for a considerably greater distance shows also that it is not merely a surficial and linear mass. It must have considerable extension in depth. In these indications it agrees therefore with the more precise solution of limiting depth given for the Texas-Kansas region.

Indeterminate evidence from anomaly contours.—The map of anomaly contours shown in Fig. 5, Part II, and reduced from Bowie, does not in general throw positive light on the depth of the masses which produce the anomalies of gravity. The necessarily generalized and smoothed-out character of this map has been discussed previously, especially in Part IV. A map based upon more numerous observations would show higher values of maximum anomaly and more of them. The centers of outstanding mass and the anomaly gradients would become better defined, and the distances from epicenter to half value of Fv would average smaller than shown at present. However, notwithstanding the defects, thirty-two measurements were made on this map of the distances from fifteen pronounced maxima to the anomaly contour of half value, and in directions not toward other adjacent maxima. This distance was chiefly controlled therefore by the single dominating mass. The measurements gave an average distance of 120 km.

If the outstanding masses which gave these anomalies were assumed to have the form of spheres, this would give their centers a depth of 160 km. and imply the existence of marked heterogeneity extending below the zone of compensation as given by Solution H. If the average form were assumed, however, to be that oblate mass shown in Fig. 9C, this distance to the contour of half value would correspond to a depth of 100 km. But such assumptions as to form are hypothetical and justifiable only as a step in successive

approximations, not as a conclusion. The anomalies can be accounted for just as readily by an assumption of much shallower depths. The outstanding masses would then possess marked thinness in comparison with their breadth. They would be horizontally extended masses or the algebraic sum of many masses; in either case they could lie within a quarter of the depth indicated by the assumption of spherical form. The mere measurement of the mean distance from epicenter to half value of *Fv* is therefore wholly indeterminate except as regards the limits of regional compensation. In some respects, however, the present map does give suggestions. Let the attention be turned to these individual features.

A line of stations extends along the margin of the Coastal Plain from Washington, D.C., to Hoboken, New Jersey. The anomalies at the stations and their distances apart as measured on the map are as follows:

No. 22. Washington, D.C., +0.039 dyne
58 km.
No. 23. Baltimore, Md., −0.011
138 km.
No. 24. Philadelphia, Pa., +0.022
61 km.
No. 25. Princeton, N.J., −0.019
69 km.
No. 26. Hoboken, N.J., +0.024

This line of stations extends in the direction of the trend of the foundation rocks, yet the sign is reversed at every station, showing marked heterogeneity even in the direction of the strike. The average anomaly without regard to sign is 0.023, a little larger than the average for the whole United States. The average with respect to sign is +0.011. As there is only one station for each of the positive and negative masses the positions and magnitudes of the real maxima and the curves of changing anomaly are unknown. Masses in the upper half of the zone of compensation could produce these effects at these horizontal distances with but little mutual neutralization. Masses below the zone of compensation would, however, have to be very great, not only because the force decreases inversely with the square of the distance, but because masses of opposite sign whose centers are more than 120 km. deep and situated but

from 60 to 70 km. apart would largely neutralize each other in their surface effects. Furthermore, there is no notable extension of the anomaly contours shown in any direction, and more especially at right angles to the line of stations, such as would suggest the wider fields of force due to deep-seated masses. If the masses were at great depth this limitation of attraction to regions near the epicenters could be produced only by a special checkerboard arrangement of opposite masses in all directions. It may be rather firmly concluded, therefore that the anomalies of this chain of stations along a line of low topographic relief are due to heterogeneities of density within the zone of compensation.

In certain regions, as in Florida, in western New York and Pennsylvania, and in the Great Basin, occur broad areas of anomaly showing no central maximum. To some extent this is doubtless due to incompleteness of observations, but in the areas mentioned the stations are so spaced as to show that even if the map were complete there would not exist marked domes of anomaly, such as those central at Minneapolis, Minnesota, and at Lead, South Dakota. This absence of domal form of anomaly curves suggests that the disturbing masses cannot be below the zone of compensation, but should be interpreted as due to the effects of masses widely distributed in the zone of compensation. This relation is especially striking in southern Nevada. The deflection residuals in northern Utah and Nevada all turn away from this southern area of defective mass, shown in Fig. 5, Part II, of this article, as located by Hayford and Bowie. Yet within this broad area of defective mass Station No. 67 shows an anomaly of only −0.013 and some of the surrounding anomalies have actually a larger negative value. There is here then an entire absence of a broad domal form. This is the region which indicates from the least-square equations of the deflections of the vertical the shallowest compensation within the United States; and the combination of the evidence from deflection residuals and anomaly contours goes to show that the anomalies are due to departures from isostasy within that shallow zone.

An inspection of Fig. 5, Part II, shows furthermore that the centers of plus and minus attraction as located by Hayford and Bowie from the deflections of the vertical, although in general

agreement with the measurements of gravity anomalies, so far as the positive or negative sign of the center of mass is concerned, yet are not closely related to the large maxima. They are, in fact, in most cases decidedly eccentric to the anomaly contours. The scarcity of these areas is a result of incompleteness of observations, but their eccentric position and association with areas of moderate anomaly is not an error due to the reconnaissance nature of the studies. These relations indicate that the neighboring regions giving broad domal areas of anomaly are in such cases not due to the dominating control of centrospheric heterogeneity, for in that case the resultants of the deflection residuals would point over broad areas in the general direction of the epicenter of the mass. The degree of discordance between the centers of dominant anomaly and centers of dominant deflection indicates that fuller observations, would produce agreement by adding to the number of such centers. The present data suggest therefore that the areas of broad excess or defect of mass as shown by the anomaly map are due to aggregates more or less composite and shallow, so that each part influences individually to some extent the direction of the deflection residuals about it. Special combinations of masses of shallow depth with other masses below the zone of compensation could, however, also account for the effects. The data of the present map of gravity anomalies are therefore largely indeterminate, but the probabilities point toward at least the greater part of the outstanding masses lying well within the zone of compensation. In this conclusion the data agree with the other lines of evidence.

RELATION OF DEPTH OF OUTSTANDING MASSES TO HYPOTHESES REGARDING DISTRIBUTION OF COMPENSATION

The measurements of the deflection residuals are very much more detailed than are those of gravity anomalies. The evidence from them is rather conclusive that, for the regions investigated, the excesses or defects of mass which cause those residuals are situated within the zone of compensation and more especially in its outer half or third. Even if centers of outstanding mass were uniformly distributed, however, with respect to depth, they would lose influence in proportion to the square of their depth. Smaller

masses which would exert a very appreciable effect if near the surface would, in consequence, not betray their existence if situated near the base of the zone of compensation. But the larger masses which are found to exist would exert a very visible control upon the deflections of the vertical, even if their centers were at a depth of 100–200 km. The fact that such depths have not been found suggests that the larger variations from the mean density within any one earth shell tend to occur in the outer half of the zone of compensation rather than in its deeper parts or immediately below it.

As a step toward the interpretation of the evidence, let the conclusion reached in Part II of this article be accepted: that regional isostasy for ordinary relief certainly extends to a radius of 100 and probably to 150 or 200 km. Even these limits do not reach the capacity of crustal strength. Such regional limits would not in reality be subject to sharp boundaries. This agrees with the evidence of geology in showing that mountain groups of circumdenudation—those whose relief is due to erosion and not to original differential vertical movement—are upheld by the rigidity of the crust. This applies to many of the mountain groups of the Appalachians; such, for example, as the Catskills.

The fairest initial hypothesis of isostatic compensation would be then to calculate for each station the average elevation of the country within a radius of 99 km., being the outer radius of zone N, and to assume a uniform density to these limits such as is needed to compensate this area. A second trial hypothesis would be to use as the radius of regional compensation the outer limits of zone O, 166.7 km. Under these two calculations for regional compensation the Catskills would be regarded as producing deflections which should show an excess of mass at the surface of the earth. Such an erosion basin as the Nashville basin should show, on the other hand, by its deflections a surface deficiency of mass. For the hypothesis which approaches nearest to the truth, the residuals of the deflections should be small and the outstanding masses would be determined by variations of density within the crust and not of the topography upon its surface.

Under the hypothesis of local compensation as given in Solution H the excess of mass in the Catskills would show, on the contrary,

as a slightly excessive density throughout the whole zone of compensation; the Nashville basin as a slightly deficient density through the same depth. The residuals should indicate an outstanding excess and deficiency of mass respectively with the centers at a depth near the middle of the zone of compensation. But masses with centers at this depth and distance would have a very diminished maximum effect upon the residuals of the deflections of the vertical, and one largely modified by the effects of contiguous regions. Heterogeneities of density nearer the surface and not related to compensation would tend also to overshadow the error involved in the hypothesis of local compensation. It would appear then that the nature of the deflections is not very sensitive for testing the relative probability of the hypotheses of local versus regional compensation. The assistance of a computing office for trying out several hypotheses would probably bring to light, however, conclusions which would be more determinative. These statements must be regarded, therefore, as forecasts not yet subjected to the tests of computation.

In view of the preceding discussion it would seem that the deflection residuals of Solution H are chiefly of value for measuring the heterogeneities of density not related to topography, nor to the mantle of sedimentary rocks. This is especially true of the Texas–Kansas region studied in detail, for there the region is one of plains with an average elevation of about a thousand feet, and the demands of local isostasy as postulated in Solution H would call for a nearly uniform density under all this region. The outstanding masses represent in large part, therefore, real and local variations from a mean density of the continental crust.

But if masses of excess or defect of density similar to those numbered 2 and 5 of Fig. 12 were widely extended, say to a radius of 500 rather than 100 km., they would tend much more strongly to make for a local or intracontinental isostatic adjustment. They would become then not outstanding masses but in large part compensating masses. The outstanding masses represent the same kind of variations, therefore, which if more broadly extended would be in accord with an isostatic adjustment of topography to a different level. They suggest that if the zone of compensation of the continental crust be divided into three shells of 40 km. each in

thickness, the greatest variations in density take place in the outer shell. This conclusion should be regarded as tentative, however, until confirmed by wider detailed studies and more numerous examples.

Accepting for the present this tentative conclusion, how does it agree with that previously reached—that isostatic compensation in some regions appears to go notably deeper than 122 km. and that, where deep, the residuals average smaller than for the continent in general? The answer would appear to be that moderate variations of density are sufficient to account for the isostatic relations of different parts of the continent to each other and that these moderate variations may go very deep.

If the actual distribution of compensation gradually disappears with depth, the hypothesis of uniform compensation complete at a certain depth corresponds to two outstanding masses, one just above the limiting surface, 122 km. in Solution H, the other just below that surface. But these masses would largely balance each other, having opposite signs; so that they would give at the surface of the earth but little evidence of their existence. Imperfections of the hypothesis in regard to the bottom of the zone of compensation would in consequence not readily be detected by methods for determining the depth of outstanding masses.

The isostatic balance of continental crust against oceanic crust is a somewhat different problem from that of the different segments of the continent with respect to each other. Solution H requires a mean difference in specific gravity of about 0.1 to a depth of 122 km. between the crust of the average continental and average oceanic segments. The contrasts in density are therefore pronounced and go very deep. Within the continent, on the other hand, the variations in density related to isostatic compensation are comparatively small and this investigation suggests that those variations may be more largely in the higher levels of the crust.

In conclusion, the depths of the outstanding masses are seen to be related to many problems in crustal statics and dynamics. The depth determines the magnitude of the masses involved and if known will serve as a test of various hypotheses. The excesses and defects of mass departing from that mean which is demanded

by the best hypothesis will be a more accurate measure of the capacity of the rigid crust to carry without viscous yielding loads which have been borne through geologic time, hidden loads whose magnitudes in many regions appear to mask by contrast the present relief between mountains and valleys.

The measure and the meaning of the variable distribution of mass within the lithosphere constitutes an inviting field of geology, discernible in the present, but whose real exploration is a work of the future.

[*To be continued*]

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THE STRENGTH OF THE EARTH'S CRUST

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PART VI. RELATIONS OF ISOSTATIC MOVEMENTS TO A SPHERE OF WEAKNESS—THE ASTHENOSPHERE.[1]

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INTRODUCTION AND SUMMARY

In studies on the nature of isostasy it is necessary to distinguish between, first, the existence of isostasy; second, the limits of isostatic equilibrium; and third, the mode of maintenance of this equilibrium.

The first has long been known, the knowledge of the existence of some relation of density counterbalancing elevation having been gradually developed since the middle of the nineteenth century through the determination of the local deviations of the vertical as shown by the comparison of the astronomic and geodetic latitudes for the same station. This was a problem which arose in both astronomy and geodesy. It was found, when the attractive effect of the mountain regions was computed, that they did not deflect the vertical at adjacent stations as much as was to be expected from their visible masses. The phenomenon was first pointed out

[1] An abstract of Parts V, VI, and VII of this series was given at the April, 1914, meeting of the American Philosophical Society at Philadelphia under the title, "Relations of Isostasy to a Zone of Weakness—the Asthenosphere." See *Science*, XXXIX, 842.

by Petit in 1849.[1] Archdeacon Pratt of Calcutta showed a few years later that whereas a discrepancy of 5.2″ existed between the geodetic and astronomic latitudes of Kalianpur and Kaliana, the calculation of the effect of the Himalayas called for a difference of 15.9″.[2]

These facts were definitely formulated into a theory of isostasy by the Astronomer Royal of Great Britain, G. B. Airy, within a year following the appearance of Pratt's paper,[3] though it remained for Dutton to recognize the large geologic significance and to coin for the relations of elevation and density the word isostasy.[4] Following this Putnam and Gilbert showed by gravity measurements that a considerable degree of regional isostasy existed over the United States.[5] Since then has appeared the much more detailed work of Hayford and Bowie, the computations made by the computing office of the United States Coast and Geodetic Survey under their directions making possible this present investigation.

Thus there has developed through more than half a century evidence beyond controversy which shows that the earth's crust in its larger relief and, within certain limits, even its smaller features, such as the great plateaus and basins, rests more or less approximately in flotational equilibrium.

The second division of the larger problem of isostasy, that of the areal limits and degree of perfection of isostatic adjustment, is the subject which has been dealt with in the previous parts of this investigation. It has been found that, although the relations of continents and ocean basins show with respect to each other a high

[1] "Sur la latitude de l'Observatoire de Toulouse, la densité moyenne de la Chaine des Pyrénées, et la probabilité qu'il existe un vide sons cette chaine," *Comptes rendus de l'Acad. des Sc.*, XXIX (1849), 730.

[2] "On the Attraction of the Himalaya Mountains and of the Elevated Regions beyond Them, upon the Plumbline in India," *Phil. Trans. Roy. Soc.*, Vol. CXLV (1855).

[3] G. B. Airy, "On the Computation of the Effect of the Attraction of Mountain Masses as Disturbing the Apparent Astronomical Latitude of Stations in Geodetic Surveys," *Phil. Trans. Roy. Soc.*, Vol. CXLV (1855).

[4] "On Some of the Greater Problems of Physical Geology," *Bull. Phil. Soc. Wash.*, XI (1889), 53.

[5] *Bull. Phil. Soc. Wash.*, XIII (1895), 31–75.

degree of isostasy, there is but little such adjustment within areas 200 to 300 km. in diameter, or of limited differential relief. Individual mountains and mountain ranges may stand by virtue of the rigidity of the crust. Even under the level plains equally great loads are permanently borne, loads produced by widespread irregularities of density not in accord with the topography above. Isostasy, then, is nearly perfect, or is very imperfect, or even nonexistent, according to the size and relief of the area considered.

The third division, the mode of maintenance of isostasy and its bearings on problems of the crust, remains to be considered. This condition of isostatic equilibrium exists at present in spite of the leveling surface actions and compressive crustal movements of all past geologic time. There must be, consequently, some internal mode of restoring more or less perfectly an isostatic condition, either by frequent small movements, or by more infrequent and larger ones.

Erosion and sedimentation result in a lateral transfer of matter, and to maintain isostasy there must be some lateral countermovement in the earth below, but in regard to how or where or when this is done, and as to what are its effects, there has been no unanimity of opinion, nor convincing demonstration.

In considering the problems of crustal dynamics some authors have regarded earth shrinkage and consequent tangentially compressive forces as controlling the nature of diastrophism, including movements of both orogenic and epeirogenic character; others, the advocates of extreme isostasy, have thought to see even in folding only the secondary effects of movements maintaining isostatic equilibrium. The first point of view emphasizes the strength and elasticity of the crust, with long-deferred periodic discharge of stress. The second point of view calls for an interpretation based on the weakness and plasticity of the crust, with resulting nearly continuous small movements restoring the delicate vertical balance destroyed by gradational actions. To what degree are the two points of view compatible and within what limits is each dominant? The problem of this chapter involves, therefore, not only the mode but the limits and effects of the movements which more or less completely maintain or restore isostasy.

The method of attack is largely one of exclusion. By showing what hypotheses cannot apply, the way is prepared for conclusions in better accord with the fields of fact and theory.

The results show that conditions of isostatic equilibrium cause the light and high segments to press heavily against the adjacent lower and heavier ones, most heavily above. The tendency is consequently for the high areas to spread with a glacier-like flow over the low areas. This tendency, however, is effectively resisted by the strength of the crust. Upon the disturbance of equilibrium by erosion and deposition there are two kinds of stresses produced which tend to restore equilibrium. The first is a tendency of the heavy column to underthrust the lighter, but it could never produce compression and folding at the surface. This force would be most effective under the hypothesis of great crustal weakness, so that the vertical stresses could be transmitted in a horizontal direction within the lithosphere as in a fluid. Even in that case, however, it would not be the dominating force. The actual isostatic movements consist of a rising of the eroded areas, a sinking of those which are loaded. This involves shear or flexure around their boundaries. The columns must be large enough so that the excess or deficiency of mass can become effective in producing deformation. When the accumulating vertical stresses have overcome the strength of the crust, the excess pressure from the heavy area is transmitted to the zone below the level of compensation. This deep zone is in turn the hydraulic agent which converts the gravity of the excess of matter in the heavy column into a force acting upward against the lighter column and thus deforms the crust of the eroded area. By this means even the continental interiors are kept in isostatic equilibrium with the distant ocean basins. This implies a great depth and thickness to the zone of plastic flow. Although it must be plastic under moderate permanent stresses, this does not imply by any means a necessarily fluid condition, and fluidity is disproved by other lines of evidence.

The zone of compensation, being competent to sustain the stresses imposed by the topography and its isostatic compensation, must obey the laws pertaining to the elasticity of the solid state and is to be regarded therefore as of the nature of rock. Consequently there may be extended to all of it the name of the litho-

sphere, even though it includes from time to time molten bodies, the constituents of the pyrosphere.

The theory of isostasy shows that below the lithosphere there exists in contradistinction a thick earth-shell marked by a capacity to yield readily to long-enduring strains of limited magnitude. But if such a zone exists it must exercise a fundamental control in terrestrial mechanics, in deformations of both vertical and tangential nature. It is a real zone between the lithosphere above and the centrosphere below, both of which possess the strength to bear, without yielding, large and long-enduring strains. Its reality is not lessened because it blends on the limits into these neighboring spheres, nor because its limits will vary to some degree with the nature of the stresses brought upon it and to a large degree by the awakening and ascent of regional igneous activity. To give proper emphasis and avoid the repetition of descriptive clauses it needs a distinctive name. It may be the generating zone of the pyrosphere; it may be a sphere of unstable state, but this to a larger extent is hypothesis and the reason for choosing a name rests upon the definite part it seems to play in crustal dynamics. Its comparative weakness is in that connection its distinctive feature. It may then be called the sphere of weakness—*the asthenosphere*, and its position among the successive shells which make up the body of the earth is as follows:

The atmosphere The hydrosphere	Including the biosphere
The lithosphere The asthenosphere	Including the pyrosphere
The centrosphere, or barysphere	

Each has played its fundamental part in the development of earth-history.

STRESS-DIFFERENCES BETWEEN CONTIGUOUS COLUMNS OF THE CRUST

Stresses under conditions of isostatic equilibrium.—The continental platforms slope down into the ocean basins at grades which range mostly from one in ten to one in thirty. Some of the great

foredeeps show both the greatest depths of water and the steepest descents. The Chilean coast, for instance, at lat. 25° S., slopes from the Andes to a depth of 7,500 meters with a submarine grade of one in eight. Under the hypothesis of nearly perfect isostasy, which will be favored in this discussion, this would be taken to show the contiguity of areas in the crust of markedly unlike density.

Let the slope between such areas be regarded as a thick partition between two columns, each in isostatic equilibrium. These rest then upon the substratum below the zone of compensation with the same pressure and stand vertically in equilibrium.

In so far as the rock within the crust is subjected to mere cubic compression, equal in all directions and increasing with depth, there is no distortional force. In so far, however, as side pressures in one column are not balanced by equal side pressures from the adjacent columns, there is a stress-difference which does produce a distortional strain. If the stress-difference exceeds the elastic limit a permanent deformation results which reduces the stress and eases the strain. It is the plan of this paper to discuss the nature of the stress-differences on the partition separating two contiguous columns of the crust, of markedly unlike density; first, when these are in isostatic equilibrium, and second, when not in such equilibrium. Fig. 13 is drawn to show graphically these relations.

The land-column of the crust is marked *M;* the submarine column is *N; O* is the earth-shell below the zone of isostatic compensation; *P* is the column of sea-water. The vertical partition between the unlike columns stops in reality, according to the hypothesis, at the bottom of the columns. It is here extended down through the earth-shell *O–O* in order to discuss the deformation which would take place in the latter shell. *M* and *N* represent what is here called the lithosphere; *O–O* the zone which it is proposed to call the asthenosphere.

In case A, isostatic equilibrium is assumed and the pressures of the two lithospheric columns are equal upon the asthenosphere. But, assuming for the moment that the vertical pressures are freely transmitted as lateral pressures, it is seen that a marked horizontal unbalanced pressure is produced by the land-column against the

sea-column, as represented by the horizontal lines of the stress diagram. The top of the land-column is balanced only against the negligible weight of the atmosphere and the lateral stress gradient is there highest. The next portion below is balanced against the

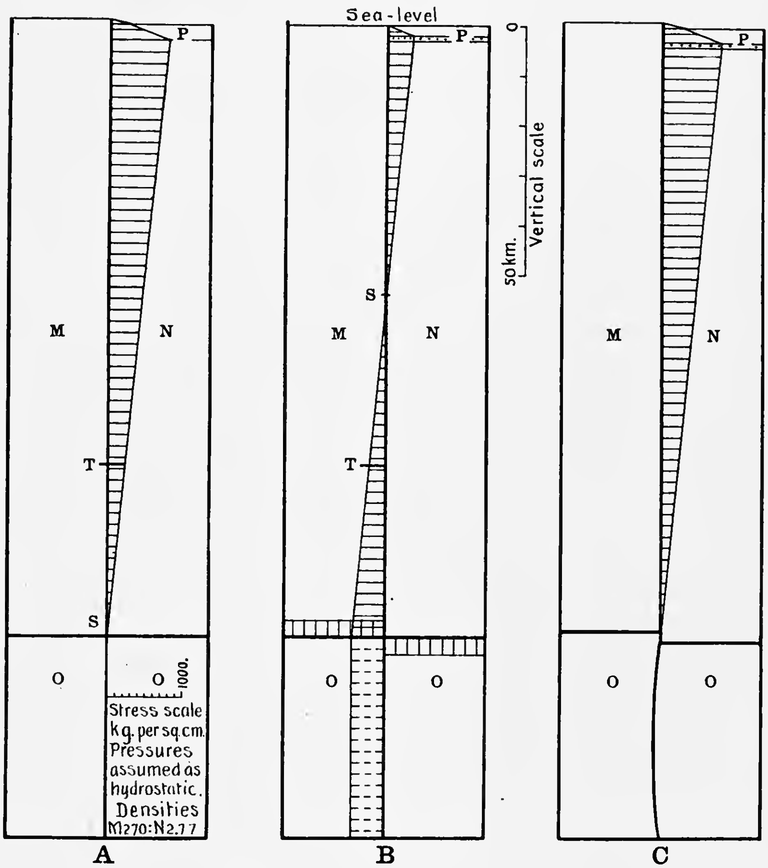

FIG. 13.—Diagram illustrating pressure-relations of the crust for marginal portions of the continental shelf and oceanic basin, interpreted as balanced by uniformly distributed isostatic compensation. Stress-differences are shown by cross-lined diagrams, the pressures being regarded as transmitted hydrostatically. The actual lateral stress-differences, for stresses within the elastic limit, are about one-fourth of the hydrostatic pressures here shown.

A. Columns in isostatic equilibrium.
B. Relations after base-leveling.
C. Relations after re-establishment of isostatic equilibrium.

sea-water and the stress gradient becomes less high. The maximum thrust occurs at the bottom of the ocean and is from the land toward the sea. Below this level the density of the sea-column is greater than that of the land-column. This, with increasing depth, gradually balances the excess pressure, and at the base of the lithosphere both the lateral and vertical pressures of both columns by hypothesis are equal.

In this diagram the pressures of the columns are imagined to act hydrostatically, but, in reality, for stresses *within the elastic limit*, this would not be so. Further, in so far as the partition is much wider than the difference in elevation of the columns, it has a gentle surface slope and will tend to give the upper part of the land-column competence to hold itself in by its own strength and that of the partition. The approximate ratio which the actual lateral pressure-differences on the two sides of the partition hold to the assumed hydrostatic pressures may be perceived from the results of a recent work by Love entitled *Some Problems of Geodynamics*.[1] In chaps. ii and iii he considers the problems of the isostatic support of continents and mountains. As a basis for the analytic treatment he assumes, first, the existence of complete compensation within a depth of one-fiftieth of the earth's radius, 127 km.; second, that at this depth all stress-differences disappear, the pressures below being of the nature of hydrostatic pressures, the only kind which could occur if a fluid layer existed at and below the depth of 127 km.; third, it is known that the heterogeneities of mass in the lithosphere only slightly modify the form of the geoid, and it is accordingly assumed that there is no such effect. Love thus treats of the limiting case of a crust exhibiting perfect isostasy, its surface relief not modifying the form of the geoid given by the ocean surface, and resting with its base upon a fluid zone. As such, his solution is of great value, but he states: "It must, however, be understood that the special form (of the hypothesis of isostasy) is introduced for the sake of analytical simplicity rather than physical appropriateness."[2]

The artificiality of the assumption of the existence of no stress-differences below the zone of compensation is shown by the law of

[1] Cambridge University Press, 1911. [2] *Op cit.*, p. 7.

density distribution which results. With only these three limiting assumptions, the number of unknown quantities remains larger than the number of equations, and the results are, strictly speaking, indeterminate; but by making various reasonable further assumptions definite solutions in accordance with these may be obtained. The elimination of stress-differences at the base of the lithosphere, taken as equivalent here to the zone of compensation, requires, however, that there shall be a peculiar relation of densities. To compensate an elevation it must be offset by matter below of less density than the mean for that depth, but in order to quench the stress-differences at the base of the lithosphere there must be between the light matter and this base a layer of more than mean density for that depth. Thus the light layer must perform a two-fold function, compensating not only the elevation above but the heavy layer below. For depressions in the crust there must be a reverse arrangement, matter of more than mean density existing immediately below the surface. But above the base of the lithosphere there must be a layer of less than mean density for that depth. The artificialities of this scheme would be sufficient to form a disproof of the initial assumption which determined it, but it also seems to be directly disproved by the evidence brought forward in the earlier parts of the present article. Nevertheless, the exact mathematical solution of this difficult problem is of great value as giving the results of the assumptions of extreme isostasy.

For the largest inequality of the crust, regarded as a zonal harmonic of the first order, that represented by the land and water hemispheres, Love shows that the lateral stress-differences under this hypothesis of isostasy reach a maximum at a depth equal to one-third of the zone of compensation and are equal to only 0.006 of the weight of a column of rock of height equal to the maximum height of the inequality. For harmonics of the second and third orders, representing the continents, the fractions are 0.0134 and 0.0208. These results, Love states, are extremely favorable to the hypothesis of isostasy, since the inequalities could be supported by any reasonably strong material.

There are two criticisms, however, to be noted while citing this conclusion. First, it is known that the crust is vastly stronger than

these requirements, so that such a perfected isostatic arrangement is not demanded on the score of crustal weakness. Second, the harmonic curves giving these figures are of a gently sweeping character; whereas, the actual continents are in many places high on their margins, and from these margins they slope with comparative steepness to the mean depth of the ocean floors. The stresses set up beneath the continental margins are accordingly a closer approximation to those imposed by lofty mountain ranges. Assume that compensations of the continental margins are perfect and the problem becomes that which Love takes up in the following chapter, namely, the isostatic support of mountains, except that we deal with only one great slope, whereas the theory calls for a succession of mountains and valleys.

It is shown that for such a compensated series, postulating the distribution of densities previously discussed, the greatest stress-difference exists at the mean surface, beneath the crests, and reaches a value equal to half the weight of a column of rock equal to half the height of the crests above the valley bottoms. From this maximum the stress-difference decreases to zero at the base of the zone of compensation. The solution by G. H. Darwin for uncompensated mountains and valleys gave a maximum stress-difference equal to 74 per cent of half the height, this maximum occurring at a depth equal to about one-sixth the distance between mountain crests. Even with perfect isostatic compensation, distributed after the fashion assumed by Love, the stress-differences for mountains and valleys are seen consequently to be two-thirds in value of those produced by an uncompensated relief, and are approximately one-fourth of the hydrostatic pressures. This fraction, one-fourth, happens also to be the same as Poisson's ratio, the ratio of the lateral expansion to the vertical shortening of a free rock column under vertical stress.

Now the distribution of density has been found to be more or less irregular, and there is no evidence of such a reversing layer at the base as Love has postulated. Stress-differences will consequently extend below the isostatic compensation. If, however, the latter is not uniformly distributed, but is concentrated somewhat in the outer half of the lithosphere, the stress-differences will become

small at and below the base of the lithosphere. On account of the incompleteness of local compensation, the irregularities and uncertainties of the actual facts of nature, the Gordian knot of a solution may be cut by simply assuming for present purposes the form of diagram given by hydrostatic pressures due to a compensation uniformly distributed. The approximate stress-differences will be given by taking one-fourth of the values given by the hydrostatic pressures. This transfers the problem from the difficult field of zonal harmonics to the simple one of hydrostatics, and perhaps does not introduce errors greater than those involved in the differences between nature and the postulates which form the foundation of the solution by zonal harmonics. This hydrostatic diagram is shown accordingly in Fig. 13.

It is held by the advocates of extreme isostasy, however, that for long-continued stresses the crust is very weak; in other words, the elastic limit is low, and slow plastic deformation readily occurs which tends to dissipate the stress-differences and re-establish isostatic equilibrium. To the extent to which this is true, the real diagram of lateral stresses would approach the hydrostatic diagram here given and measure the forces producing plastic flow.

It has remained, however, for the opponents of the hypothesis of local and nearly perfect isostasy to point out, what is here illustrated graphically, that the extreme theory requires a belief in vertical weakness but lateral strength. If it were not for lateral strength the land-column would crowd against the sea-column, more at the top than at the bottom. Flowing out with a glacier-like motion over the upper part of the sea-column, the land-column would settle at the top and become shorter. This in turn would bring about a vertical elevatory pressure against its bottom, the column would rise, lateral creep would continue with equal pace, and the end result would be a density stratification in which the continental crust would come to overlie the oceanic crust. The limit of such an action would be given by the decreasing surface gradient, this finally becoming so gentle as to stop the glacier-like flow. The lack of such an effect implies of course that the lateral stresses of the outer part of the lithosphere lie within the elastic

limit. Therefore they may be regarded as having not more than a quarter of the value shown in Fig. 13A.

The suggestion of the existence of opposing modifying factors is to be found in conclusions from the previous parts of this investigation—that compensation may be in many places concentrated somewhat in the outer half of the zone as here shown and in other places fade out through a notable distance below. These two variations in the distribution of compensation would modify the stress diagram in opposite directions.

Modifications of stresses produced by base-leveling.—Consider next the case of complete erosion to sea-level, as shown in Fig. 13B. The rock from the land-column has been deposited as sediment over the sea-column. As the columns are supposed to act as units the sediment is shown as spread uniformly. The lateral stress diagram beneath the bottom of the sediment shows a rate of decrease the same as in case A, but the value of the hydrostatic stress at any depth is diminished by the sum of the depths of erosion and deposition. The lateral stress now changes in sign at a point *S* and at this depth is a line of no lateral stress. Above this depth the continental segment tends still to spread over the ocean, but less effectively than before; below this depth the oceanic segment now thrusts against the continental crust.

If the ocean water be eliminated from the diagram and base-leveling should bring both columns to a uniform surface, then the neutral depth *S* advances to the surface and the lateral stress diagram in B is just the reverse in value to A. In that case there is no lateral thrust at the surface, but at all depths below there is an excess pressure against the continent reaching a maximum at the bottom of the lithosphere. This extreme case cannot apply to the ocean except for that limited width over which is built out a continental shelf. To the degree to which the weight of this shelf is supported by the ocean crust beyond, the column beneath the shelf would not operate with its full pressures against the land. The case would apply better to the complete erosion of level-topped plateaus situated within a continent.

For the lateral pressure within the lithosphere to become effective in a landward undertow would require a lesser rigidity of the

crust at the bottom than at the top. Such a lesser rigidity may be granted, but it is seen then that the landward undertow would be greatest at the bottom and could not advance above a depth indicated on the diagrams by *T*. At this point the stress is of the opposite sign but of the same value as for the state of isostatic balance in case A. If seaward flow did not take place at this level in the first case, landward flow could not take place in the second.

For the extreme case where isostasy is completely destroyed by surface leveling, no water body remaining, *T* will rise upward to a depth equal to one-half the depth of the zone of compensation. If the surface of complete compensation be 76 miles deep, this gives a minimum depth of 36 miles. For the undertow to reach this height implies, however, not only the limiting case of complete destruction of isostasy, but a crust only one-half as rigid at depth *T* as at the surface and a previous state of expansive surface stress as great as the outer crust could bear. On the other hand, if tangential pressures due to centrospheric shrinkage should co-operate with the stresses tending to restore isostatic equilibrium, underthrust would become more effective below, but overthrust would also become effective above.

The disappearance of isostatic compensation at a certain level means the disappearance of notable heterogeneity in the earth-shell below, as argued in Part V (pp. 446–48). One of the possible suppositions to explain this is to suppose that this shell is weaker than the crust above and therefore the lateral thrust due to an assumed initial heterogeneity would cause a lateral flow, a density stratification, and a resulting disappearance of the postulated heterogeneity. This supposition of a weaker zone finds support in other lines of evidence. Therefore, although some lateral flow at the base of the lithosphere may occur during the restoration of isostatic equilibrium, it is to be expected that the bulk of such flow will be below, for there the substance is more plastic and the lateral stress is throughout at a maximum.

Let attention be given next to the vertical as contrasted to the lateral unbalancing brought in by the destruction of isostatic equilibrium. The land-column becomes lighter, the sea-column heavier, by amounts which are shown in the *vertically lined* stress

diagrams at the base of the lithosphere in case B. Supposing that vertical readjustment of the columns is prevented for a time by the strength of the crust, the vertical stresses will be taken up by a vertical shearing strain along the partition between the two columns. This shear is equal in amount to the difference in total weights of the columns. Let the shear per unit area be called s. It acts over a surface taken as 122 km. high. Let this height be called h. The weight of the columns will vary with their breadth in the plane of the drawing. If the breadth of each be taken as b and the weights per unit area as M and N (N including rock, sediment, and sea-water), then for a cross-section of unit thickness the total difference in weight is $(N-M)b$ and the total shear is this same amount, provided that the columns are not sustained in part by other boundaries. But the total shear is also sh. Therefore

$$sh=(N-M)b$$

$$s=(N-M)\frac{b}{h}$$

For narrow columns b is small, giving to s a small value and consequently one within the elastic limit. Let b become broad and s will then become large and exceed the elastic limit. The *lateral* pressures, on the contrary, are less dependent upon the breadth, and, if the problem were regarded as one of hydrostatic pressures, would be wholly independent of breadth. The formula shows that the broader the columns, the more readily they will readjust by vertical shear between the columns. Now unless failure by vertical shear took place between the upper part of the columns the heavy column would be held up, the light column would be held down, except for the partial effect of sagging in case the columns were very broad. The lateral landward pressure at the base could therefore not become effective. The loaded portion of the crust must fail first by shear or flexure of its upper portion. Whatever be the distribution of strength it would appear then that the primary yielding is the vertical one and the landward force of undertow can become only secondarily effective.

The hypothesis of local and nearly complete isostasy requires that the elastic limit for vertical shear should be very low in order

that narrow columns should be able to rise or sink. This may be illustrated by the following example:

Suppose a region 50 km. in radius possesses a mean departure from isostatic equilibrium equal to 76 m. of rock (250 ft.) and that the surrounding regions are out of adjustment by the same amount but in the reverse direction. This is the maximum area for regional isostasy which in Hayford's opinion is to be expected, and 250 ft. is the mean departure from isostasy as given by him in his Minneapolis address. But in this example the adjacent regions are each assumed to be out of adjustment in opposite directions by this amount and, therefore, the differential load is twice this or 500 ft. of rock. The case is one which he would regard consequently as rather extreme. Now a cylinder 100 km. in diameter and 122 km. deep could not fail through its bending moment, as in the flexing of a beam. It would have to fail as in punching a rivet hole through a metal plate, in other words, by circumferential shear. The shearing stress per unit area is obtained by dividing the total load by the total shearing surface. With the data taken as above this gives $s=8.4$ kg. per sq. cm. or 120 lbs. per sq. in. But strong rock at the surface can readily carry a shearing stress of from 700 to 1,000 kg. per sq. cm. (10,000 to 14,000 lbs. per sq. in.). Isostatic perfection to this degree would therefore require the zone of compensation as a whole to be only about one-hundredth as strong under permanent stress as is solid rock at the surface. This calculation alone would tend to show that the loads and areas by which the crust departs from isostatic equilibrium have been much underestimated by the advocates of extreme isostasy.

It should be noted, however, that, following the lines of his rejoinder to Lewis, Hayford would answer that he regarded the landward isostatic flow as taking place within the zone of isostatic compensation and the vertical shear as operating, consequently, through a depth far less than the thickness of the entire zone of compensation. There are, however, a number of inconsistencies in this argument, some of which have already been made evident. Others will appear as a result of the later discussion of this chapter. But it may be noted that even granting this contention—that only the outer third of the zone of compensation was involved—the

unit shearing stress would be multiplied only by two or three and would still imply a weakness in this part of the crust to resist long-enduring shear or bending stresses, its capacity being only 3 or 5 per cent at most as great as is found to exist in surface rocks for stresses of human duration.

Relief of stress accompanying restoration of isostasy.—It is seen from the preceding analysis that the movement of the unbalanced columns toward a new state of equilibrium will be partly by vertical shear in the neutral ground between them, but, where the areas are large in comparison with the thickness of the zone of compensation, the easiest mode of yielding may be by flexure, showing at the surface as crustal warping. Both modes of yielding serve to transmit the excess vertical stresses of the heavy and sinking column into the asthenosphere. If the latter be indeed a shell of weakness it will transmit these pressures more or less hydrostatically. The vertical pressure-differences will act within it as lateral pressures making for flow toward the lighter column. It is shown in Fig. 13B that the maximum horizontal stress in so far as it approaches a hydrostatic distribution acts throughout the whole depth of this zone, so that it not only is weaker than the crust above, but is subjected to maximum stress over a greater area. It will yield by flowage therefore either if of small depth and very plastic, or of great depth but more rigid. If the columns are adjacent and narrow as compared to the thickness of the shell of weakness, then the principles of plastic flow would require that the flow be chiefly in the upper part of this shell. If, however, the columns are of considerable breadth compared to the thickness of the asthenosphere, and especially if at a distance from each other, then the principle of least work would determine that the middle strata of this shell should flow the farthest and the whole would to some degree participate. If an imaginary partition were extended downward through this shell as shown in A and B of Fig. 13 this partition would be found warped after the movement as shown in C of the same figure.

It was seen in an earlier part of this discussion that, even supposing deformation became effective by means of the lateral stresses within the lithosphere and without the existence of a zone

of weakness below, still only the basal part below the point T would be competent to give a landward movement during the restoration of isostatic equilibrium. But now it is seen that in the asthenosphere the lateral pressures are transmitted with greater amount, from a greater distance, and with a greater cross-section. The zone is one without notable isostatic compensation within it and is presumably more plastic than the basal part of the lithosphere. Therefore there is good reason to believe that the subcrustal undertow is restricted to the asthenosphere.

The forces actually needed to produce flowage would be in reality but a fraction of those indicated in Fig. 13B as existing in the asthenosphere. The reason is that the greater part of the vertical forces is consumed in producing flexure and shear in the lithosphere. Only a residuum is needed to produce a slow plastic flow in the shell below. For that reason broken lines are used in that part of the stress diagram. The energy consumed within the lithosphere by its deformation will be nearly independent of the breadth of the columns; it will actually tend to become somewhat less with breadth because flexure on large radii will be favored. The energy consumed in the asthenosphere will, on the other hand, increase with the breadth of the columns, but will be spread over a greater area. The temperature effect due to the absorption of energy would appear to be a minor factor, for it cannot exceed that energy which is supplied by the average vertical stress-difference multiplied by the vertical distance moved. The average vertical stress-difference will be the mean between that at the beginning of movement and that residual stress remaining after the movement is completed.

In determining the scale of the diagrams of Fig. 13 the following data were chosen. The land-column was taken in A as having a surface elevation of 1,000 m. and a density of 2.70; the sea as 3,000 m. deep, and the rock below as possessing a density of 2.77. The sea-water has a density of 1.03. These relations give an isostatic balance at a depth of 122 km. In B, erosion of the land to sea-level is supposed to have taken place and the sediment spread with same unit weight over the sea-column that it had as

rock upon the land. These relations give a depth of 54 km. to S and 88 km. to T.

It should be repeated, however, in closing this topic, that the solutions here given are approximate only and assume that isostatic compensation results in lateral stress-differences which show the same distribution of forces as a diagram of hydrostatic pressures, differing only in magnitude. The writer is inclined to think that the actual facts of nature call in most cases for some depression in depth of the critical points beyond those here shown. Especially is there likely to be under the margins of a continent in isostatic equilibrium some permanent lateral stress-difference within the asthenosphere, due to the compensation above and tending toward a landward undertow. Upon the unbalancing due to erosion and sedimentation this would cause the lateral stress-differences within the asthenosphere to rise more quickly to the low elastic limit and permit more readily than would otherwise be the case a regional readjustment toward isostasy.

RELATIONS OF UNDERTOW TO THE ZONE OF COMPENSATION

Present status of the problem.—The causes of vertical movements Dutton[1] made twofold. He clearly distinguished on the one hand between those internal forces leading to expansion or contraction, which tend, by producing changes in density, to create isostatically a new surface relief, and, on the other hand, those isostatic readjustments following erosion and sedimentation, readjustments which tend not to make a new, but to restore the older, relief. Folding he regarded as unrelated to the former, as a result of the latter. He had shown earlier (in fact, he had the honor of being the first to show) that the time-sanctioned hypothesis of cooling as a cause of crustal shrinkage and consequent mountain-making was inadequate to account for either the distribution or amount of folding.[2] From this he was led to regard folding as due, not to any kind of contraction, but as a compressive movement of one section

[1] "On Some of the Greater Problems of Physical Geology," *Bull. Phil. Soc. Wash.*, XI (1889), 51–64.

[2] C. E. Dutton, "A Criticism upon the Contractional Hypothesis," *Am. Jour. Sci.*, VIII (1874), 113–23.

of the crust against another, presumably offset by tension in some other region. Dutton's argument is that the crust beneath the plateau is unloaded by erosion, that the crust beneath the basin is loaded by sedimentation. An isostatic movement, rejuvenating the relief, must, by causing the overloaded basin to settle, produce a squeezing-out of matter beneath the sinking area, and a crowding-in of matter beneath the rising area. The surficial movement of sediment is from the high area toward the low. The deep-seated movement is from the low toward the high. Thus the cycle becomes completed and the mass of matter above the level of complete compensation remains the same in each column. The seaward movement of the sediment, as a frictional resistance against the river bottoms, produces only an insignificant drag, but the return subterranean movements by viscous or solid flowage must produce a pronounced drag upon the crust in the direction of the rising region. Dutton's reasoning is clear, but the effectiveness of the action rests upon several assumptions. First, it omits the influence of the surface relief and the degree to which that tends to a lateral spreading movement from the high toward the low regions. Secondly, it postulates a low rigidity to the crust, as he in fact notes. Thirdly, it involves the conception of a strong undertow fairly near the surface in order that the crust above may be too weak to resist the viscous drag. As there were little quantitative data available at the time when Dutton formulated this corollary of his theory of isostasy he could not have tested the validity of these assumptions, but raised the problem for those who should come after him.

This theory of folding took a somewhat different form in the mind of Willis, as expressed in the concluding chapter of his *Research in China*.[1] This work in many ways is of the very first importance and gives a comprehensive view of the geological history of the whole continent of Asia. As to the nature of the movements, he finds that the continent of Asia may be resolved into positive and negative elements, the former areas tending to stand high, the latter tending to stand low. These tendencies are latent during comparatively long periods of quiet and resultant peneplanation,

[1] Vol. II (1907), Carnegie Institution of Washington.

but become operative during epochs of diastrophism. The compressive movements, on the other hand, have pressed and welded the positive elements together, the axial directions of folding representing the compression of the negative zones lying between.

The cause of the diastrophism Willis ascribes to differences in specific gravity, restricted, according to Hayford's determination, to the outer hundred miles of the earth's body; the vertical movements being chiefly due to isostatic readjustment between the several continental elements, the compressive movements being due to the tendency of the heavier oceanic segments of the earth to spread and underthrust the outer portions of the whole continental mass. This theory of the cause of lateral compression was discussed by the present writer in a review of Willis' work,[1] and the objections stated against it there are in part the same as will be elaborated farther on in the present article.

Hayford took up the same subject in his address, delivered at Minneapolis on December 29, 1910, as retiring vice-president of Section D (Mechanical Science and Engineering) of the American Association for the Advancement of Science, the title of his paper being "The Relations of Isostasy to Geodesy, Geophysics, and Geology."[2] This is a paper of broad scope intended to show how vertical movements not in apparent accord with isostasy and also movements of folding may be explained as secondary results of isostatic adjustment and really in harmony with the hypothesis of nearly continuous movement in a crust of low rigidity and of almost complete isostasy. This part of his theory is essentially the same as Dutton's but is elaborated in greater detail.

Harmon Lewis called attention to the defects in this theory of deformation,[3] but Hayford made a rejoinder, positive and sweeping in its style, to this and other lines of criticism by Lewis.[4]

The names of Dutton, Willis, and Hayford deservedly carry much weight and must be accepted at their face value by geologists

[1] *Science*, N.S., XXIX (1909), 257–60.

[2] Published in *Science*, N.S., XXXIII (1911), 199–208.

[3] "The Theory of Isostasy," *Jour. Geol.*, XIX (1911), 620–23.

[4] John F. Hayford, "Isostasy, a Rejoinder to the Article by Harmon Lewis," *Jour. Geol.*, XX (1912), 562–78.

who have not themselves made a critical study of the problems of isostasy. The arguments which the writer advanced in 1909 against this hypothesis were published under a title which apparently did not call attention to them. The style of Hayford's reply to Lewis is crushing and conveys the impression that Lewis has been completely refuted. It is because of these reasons that the subject calls here for fuller development.

In his Minneapolis address Hayford outlines a theory of the principles of diastrophism which turns upon his conclusion that isostasy is so nearly complete that areas of even limited size average only 250 feet from the level of isostatic equilibrium. He assumes chemical and physical changes to be induced in the crust by the changing load due to erosion and sedimentation. These he thinks are superimposed upon the effects of nearly continuous vertical movements of isostatic readjustment. The vertical movements in turn produce a lateral undertow which is given as a cause of localized heating and folding. Apparently this is regarded as a complete mechanism of deformation since the author raises the query:

> Is it at all certain that under the influence of such actions the geological record at the earth's surface at the end of fifty to one hundred million years would be appreciably less complicated than the geologic record which is actually before us? I think that it would be fully as complicated as the actual record.[1]

This theory of folding as the result of subcrustal undertow is illustrated by means of two diagrams. In Fig. 1, the zone of viscous flow from the sinking toward the rising area is placed in the lower quarter of the zone of isostatic compensation. In Fig. 2 it is shown in the middle of that zone, dying out both above and below. Apparently then, as shown by these two different conceptions, the author cited was guided by no definite theory, based upon the mechanics of materials, as to the factors which would determine the depth of this zone of undertow and its relations to the zone of compensation.

Harmon Lewis in his paper on the "Theory of Isostasy" has discussed various aspects of the isostatic theory as developed by

[1] *Op. cit.*, p. 206.

Hayford, and among them this question. Regarding the possibility of folding by means of isostatic undertow, Lewis concludes:

> Now, according to the theory of isostasy, compensation would be essentially complete, and if compensation is complete the depth of compensation as determined by Hayford's geodetic work would be as great as 60 miles. Hence, the undertow postulated by isostasy would exist chiefly below 60 miles. It is decidedly questionable that an undertow even much nearer to the surface than 60 miles would cause the observed folding in the upper few miles of the crust.[1]

In regard to this criticism by Lewis concerning the cause of folding, Hayford states in reply:

> On pp. 621–22 Mr. Lewis sets forth the argument that there is much geological evidence of horizontal movements in the outside portions of the earth, especially in the form of folding, that the controlling movements of isostasy are assumed to be vertical and hence cannot account for folding, and that the horizontal movement or undertow concerned in isostatic readjustment must be below the depth of compensation and hence so far below the surface as to be very ineffective in producing folding.
>
> There are two fatal defects in this argument as applied to controverting anything that Hayford believes or has written.
>
> First, Hayford has already indicated clearly his belief that the undertow concerned in isostatic readjustment is above, not below, the depth of compensation. In both the figures published in his Minneapolis address the undertow is clearly indicated as being above the depth of compensation and it is also so indicated in the corresponding text. As Hayford puts the undertow comparatively near the surface, where it is conceded that it would be effective in producing folding, the existence of extensive folding is a confirmation, not a contradiction, of his theory of the manner in which isostatic readjustment takes place. It is certainly not fair to hold Hayford responsible, either directly or by inference, for any theory which someone else may believe which involves an undertow situated entirely below the depth of compensation. Mr. Lewis apparently believes such a theory.
>
> Second, the movements which produce isostatic readjustment are necessarily horizontal, not vertical. If two adjacent columns of the same horizontal cross-section extending from the surface to the depth of compensation have different masses the readjustment to perfect compensation must involve a transfer of mass out of one column, or into the other, or from one to the other. In any case the transfer must be a horizontal movement. Hayford has already shown in print more than once that he understands that vertical movement alone does not produce isostatic readjustment. Moreover, a careful reading

[1] *Op. cit.*, p. 622.

of his Minneapolis address will certainly show that he believes that the total amount of material moved horizontally during isostatic readjustment, and especially the total number of ton-miles of such movement, is vastly in excess of the corresponding quantities concerned in the vertical components of the movement which takes place. Hence the folding and other abundant evidence of past horizontal movements observed by geologists confirm Hayford's hypothesis as to the manner in which isostatic readjustment takes place, instead of conflicting with it as Mr. Lewis' article would lead one to think.[1]

The present writer, however, believes with Mr. Lewis in the theory that an undertow must be essentially below the zone of compensation and is incapable of producing surficial folding. The reasons have been given in part in the consideration of the stress-relations, as they would exist under the hypothesis of extreme isostasy. But there are other reasons why the subject should be discussed in further detail. One reason is that, if Lewis is right on this point and Hayford wrong, it is desirable that this should be made clear, in justice to Mr. Lewis as well as to the subject. The other reason is that here in reaching a conclusion we can advantageously pursue a method of exclusion. By showing that isostatic undertow cannot take place within the zone of compensation, for various reasons besides those discussed in the stress diagrams, we reach the conclusion that it must take place in a level below that zone. Furthermore, by noting the conditions which would hinder lateral flowage we may arrive at a conclusion as to those which must exist to greater or less degree in order to permit it.

Objections against undertow in the zone of compensation.—The pressures which occur during a state of isostasy and after the destruction of that condition have been discussed. It was seen that the pressures making for the undertow necessary to restore isostasy were greatest at the bottom, but, more especially, below the bottom of the zone of compensation. The possibility remains to be considered, however, that perhaps the distribution of the rigidity of the crust more than offsets the distribution of pressures. Suppose the middle of the zone of compensation should be very weak and the crust at and below the bottom be very strong. Then,

[1] *Op. cit.*, pp. 573, 574.

if the restoration of isostasy was deferred until assisted by strong tangential pressures due to centrospheric shrinkage, it might be held that isostatic undertow could take place within the zone of compensation and between S and T of Fig. 13B. If, furthermore, compensation should be not uniformly distributed but taken as largely concentrated in the upper part of the zone of compensation, which however is contrary to the Hayfordian hypothesis, then the forces making for undertow may correspondingly rise in the crust. For these reasons it is seen that the previous argument from the distribution of pressures is not final and that the physical conditions involved in lateral flowage must also be considered.

The only positive reason which has been advanced for seeking to place the undertow within the zone of compensation is in order to utilize its viscous drag as a cause of folding. To become effective the drag must be strong, the crust above by contrast weak and therefore thin. The crumpling pressure on the *surface* of the crust cannot be transmitted directly from the sinking area, as is shown in Fig. 13, since the thrusting force is greatest at the bottom. It must be supposed to arise from the viscous drag of the undertow. But viscosity decreases the hydrostatic head with increasing distance from the source. Therefore, to permit a viscous flow at a distance from the source of pressure implies a mobility within that level of the crust which would make it wholly incapable of carrying the stresses necessary to maintain its own isostatic equilibrium. Therefore this level, by the very terms of the general conception of isostasy, would become the bottom of the zone of compensation.

As another mechanical defect of the theory under review, it is to be noted that the section of undertow taken by Hayford as in the middle of the zone of compensation is not given as involving more than half of that zone. This is as if a viscous fluid were transmitted through a pipe in which the cross-section of pipe and fluid were equal. To assume that the fluid is free to escape into a region of less pressure at the far end and yet gives such a frictional resistance against its walls as to be able to crumple up the pipe is to assume that the two are of the same order of strength. The materials of pipe and fluid might almost be interchanged.

In such viscous flow the tendency would be for a swelling and bursting to appear at the near end rather than a through flowage with a crumpling of the pipe at the far end.

Finally, the greatest theoretical difficulty is encountered when it is sought to transmit matter from beneath the regions of oceanic marginal sedimentation to beneath the regions of a continental interior. Either directly or indirectly there must be a subcrustal transference going forward all the way between these distant regions; for example, from beneath the Mississippi and Colorado deltas to the fields of erosion in the Rocky Mountains, if a condition of even approximate isostasy is to be maintained throughout. This does not mean of course that an individual ton of plastic rock is transferred a thousand miles to balance a ton of sediment. Each subcrustal unit may be transferred only a mile, but it involves a subsurface movement of matter all the way from the regions of sedimentation to the regions of erosion.

Now this implies a *continuous pressure-gradient*, and even under the conception of great crustal weakness, a pressure-gradient which could fold the weak cover-rocks would be far higher than that needed for the movement of a continental glacier. Any large degree of viscous resistance in the zone of undertow would therefore require, in order to initiate movement, an enormous defect of isostasy under the distant continental interior, an enormous excess under the marginal oceans. After a rejuvenative movement had started, it would be slow, the frictional and deformative resistances nearly balancing the deforming force. Therefore inertia of the moving mass could not carry it appreciably beyond the point where the moving force, weakened by loss of head, would just balance the resistances to further movement. It would be expected, in consequence, that a residual pressure-difference would remain, even after a period of restorative isostatic movement. But an inspection of the map of New Method anomalies given in Part II, p. 153, does not show any such anomaly gradients as would comport with this expectation. A vast region of the continental interior extending from Lake Superior to the Rio Grande and westward to beyond the front ranges of the Rocky Mountains shows average positive anomalies, indicating an excess of matter, not a

deficiency. To the westward is a broad region of average negative anomaly reaching a maximum at centers near the Pacific coast and no marked excess is shown near the mouths of the great rivers. Such a lack of regional relations would appear to show that the anomalies are due much more to local loads and irregularities upon and within the lithosphere, and to bowings due to great compressive movements unrelated to isostasy, rather than to the existence of an isostatic gradient leading from the ocean borders to the interior fields of great erosion. Therefore either the idea of strong viscous drag by undertow or else the very doctrine of isostasy—one or the other—must be abandoned. But it has been seen that if undertow exists in a comparatively plastic stratum, then that physical condition will cause it to be the bottom of the zone of compensation. Thus the application of every pertinent engineering principle reduces the initial hypothesis of surface folding by isostatic undertow, and, especially by undertow within the zone of compensation, to an absurdity.

Undertow restricted to a sphere of weakness—the asthenosphere.—All of this accumulative argument has not been advanced merely to show that a certain view is wrong. Rather has it been the intention to prepare the ground for what would appear to be a sounder theory of the mode of maintenance of isostatic equilibrium.

As for the basis of that theory, Schweydar, from the mathematical analysis of the measurement of the tides in the crust by means of the horizontal pendulum, has found that they are in accord with the assumption of the existence of a slightly plastic zone about 600 km. thick beneath a more rigid crust 120 km. thick.[1] It would appear that the geodetic evidence of isostasy points also toward the existence of such a thick and somewhat plastic zone beneath the more rigid lithosphere. It gives no knowledge of the exact thickness or depth, but for convenience the figures given by Schweydar will be assumed. It is a matter of importance to note however that, although the quantitative limits are uncertain, the suggestions given both by the tides and by isostatic

[1] "Untersuchungen über die Gezeiten der festen Erde und die hypothetische Magmaschicht," *Veröffentlichung des k. k. Preusz. geodät. Institutes*, Neue Folge No. 54, Leipzig (1912, B. G. Teubner).

compensation point to a zone of weakness much deeper and thicker than the figures which have customarily been taken as a probable depth of origin of magmas. The latter however rests upon uncertain extrapolation, whereas the figures for the limits of the asthenosphere, although of no exactness and perhaps 20 or 50 per cent from limits which finally may be chosen, have at least been determined by more direct evidence. In such a thick shell of weakness, the readjustment, after an erosion cycle, of a continental interior to isostatic equilibrium would require but very little viscous shear and but little lateral movement.

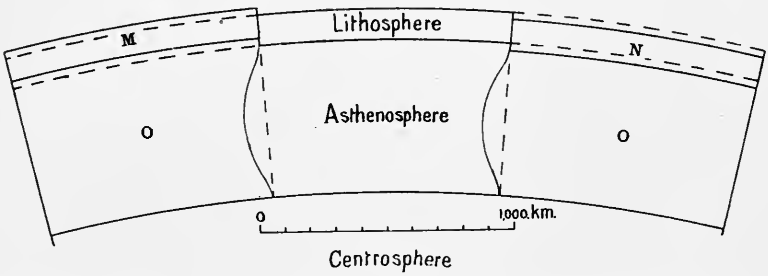

FIG. 14.—Diagrammatic vertical section of the crust, to show nature of undertow in the asthenosphere necessary to restore isostatic equilibrium in a positive interior continental area after a cycle of erosion. Effects of a vertical movement of 0.5 km. exaggerated 60 times. Asthenosphere grades into contiguous spheres and best limitations in depth are not known.

To give quantitative visualization to this conclusion Fig. 14 is drawn. Suppose a plateau area 1,000 km. wide in a continental interior to be separated from the region of sedimentary deposit by an intermediate region 1,000 km. across. Take a section 1 km. wide through these regions. Let an erosion cycle cause the removal on the average of 0.5 km. of rock from this area to be deposited over an equal area of sea-bottom. Then, during an epoch of diastrophism, assume complete recovery of isostatic equilibrium by undertow in a sublithospheric zone of weakness 600 km. thick. The vertical section of rock eroded is 500 sq. km. in area. As we have chosen a width of section of 1 km. we may also speak of this as the volume, 500 cu. km. To restore the mass of this column, 500 cu. km. must be added to it and flow past the vertical line which bounds it on the seaward side. As this zone of

flow is 600 km. deep, the actual lateral movement, if all depths move equally, will be but 0.83 km., since $0.83 \times 600 = 500$. If the flowage is supposed to increase regularly from top and bottom to the middle the movement of the middle layer would be 1.66 km. A previously vertical line 600 miles long through this asthenosphere would then be bent at the middle by this amount and its two halves make angles of $0°19'$ with the vertical. Each layer a kilometer thick would move horizontally 5.6 m. with respect to each adjacent layer of kilometer thickness. These figures bring out the insignificant degree of the plastic deformation in such a deep zone which is needed to restore isostatic equilibrium, even for a large interior continental area after erosion amounting to two-thirds of the present average elevation of the North American continent.

As a matter of fact the cross-section of the plastic deformation would not be a triangle, but a sinusoidal curve, so that the maximum linear flow for thickness of 600 km. would be between 0.83 and 1.66 km.

This illustration makes it clear that the isostatic rejuvenation of continental interiors as well as of the margins, which meets such grave difficulties under the hypothesis of a thin and shallow zone of isostatic undertow, is eliminated by adopting the hypothesis of a thick and plastic sublithospheric shell, such as has been found to be suggested by independent evidence.

The idea of folding as a result of isostatic undertow definitely may be abandoned, but the absence of a notable isostatic gradient has some further significance. It is seen from Fig. 14 that if the fields of great erosion and deposition are within a few hundred kilometers of each other the rejuvenative undertow, under the laws of stress distribution in plastic bodies, would involve mostly a limited tract in the outer part of the asthenosphere; whereas, if the undertow must extend over distances of 1,000 km. or more, then the whole depth of the asthenosphere will become involved. The amount of stress-difference and of plastic shear per unit of volume may therefore be no greater in the one case than in the other. Especially, if the middle of the asthenosphere is its weakest part, a movement generated by areas large enough to involve the whole of this zone would go forward under less stress-difference per unit

of area than for more local adjustments. The absence of a notable continental gradient is suggestive therefore of a deep zone of weakness, least resisting in its central portions, and of very marked plasticity in comparison with the rigidity of the lithosphere above. This does not involve, however, the conception of a truly fluid zone, but merely that of a comparatively plastic solid.

The existence and nature of this zone of weakness is seen to enter vitally into the theory of isostasy and must of course bear with equal importance on other branches of terrestrial dynamics as well. It is proposed therefore to elevate it to equal rank with the other shells of the earth and to name it for that quality which, from the standpoint of diastrophism, is its most significant feature as compared to the zones above and below. This is its inability to resist stress-differences above a certain small limit. Its name, therefore, is the sphere of weakness—the asthenosphere.

[*To be continued*]

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THE STRENGTH OF THE EARTH'S CRUST

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New Haven, Connecticut

PART VII. VARIATION OF STRENGTH WITH DEPTH AS SHOWN BY THE NATURE OF DEPARTURES FROM ISOSTASY

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INTRODUCTION AND SUMMARY

The first five parts of this article have concurred in showing that the crust is very strong when measured by its capacity to support great deltas, individual mountain ranges, or great internal loads due to irregularities in density not in accord with the topography. On the other hand, the altitudes of the continents as a whole, or of large sections of the continents, agree with the demands of nearly perfect isostasy. In Part VI it was shown, however, that, although even perfect isostasy threw very considerable stresses

[1] Section B, on Applications of the Theory, will be published in the following number of this *Journal*. The Introduction and Summary apply however to both sections.

upon the outer part of the crust, the maintenance or restoration of the isostatic condition through geologic time in spite of the opposing geologic activities implied the existence of an undertow below the zone of compensation. The existence of this regional isostasy for continental interiors as well as for ocean basins suggests, furthermore, that this zone of undertow is both thick and relatively very weak to resist shearing stresses. But if such a zone exists it must have important bearings on other branches of terrestrial dynamics besides that of isostasy. Its importance gives it a right to a distinct name, and it has been called here the zone of weakness—the asthenosphere. It is desirable to test its reality and its character by other lines of evidence, and such another line forms the basis of this part.

George H. Darwin has investigated the problem of the stress-differences imposed on the earth by the weights of continents and mountains. In his work the earth was assumed to possess competent elasticity throughout and the topography to be without isostatic compensation. Love has more recently treated the contrary problem of the isostatic support of continents and mountains, assuming as governing conditions that isostatic compensation was perfect within a depth of one-fiftieth of the earth's radius, 127 km., and that all shearing stresses due to topography and compensation disappeared at that depth. Below there is assumed to exist only hydrostatic pressures. In other words, Darwin postulates no isostasy and no asthenosphere; Love postulates perfect isostasy and a perfect asthenosphere. As there is known to be no truly fluid universal shell within the earth, and as isostasy for limited regions is far from perfect, the truth must lie between these two extremes. The asthenosphere must have some degree of strength and a measure of its strength is derived in this part by a study of the nature of the departures from isostasy.

For this purpose is discussed the nature of the stresses as worked out by Darwin. Then the departures from isostasy are analyzed into harmonic series. Those of long wave-length are seen to be of low amplitude, those of short wave-length of high amplitude. Now the departures from isostasy are according to their very nature without compensation and their stress effects will therefore

follow Darwin's law except in so far as the great rigidity of the outer crust will permit it to sustain loads after the manner of a continuous beam. It is shown, however, that the outer crust is inefficient as a beam, so that the results of applying Darwin's analysis will probably not be greatly modified.

It is found that the departures from isostasy are such as throw great stress-differences upon the zone of compensation, here called the lithosphere. The maximum stresses imposed by the loads found to exist within the United States lie furthermore within the outer two-thirds of that zone. The stress-differences due to this cause reach maxima probably between 3,000 and 5,000 pounds per square inch.

Within the asthenosphere, on the contrary, the stresses caused by the departures from isostasy are very small, under the United States the stress-differences at depths of from 400 to 600 km. reaching maxima probably between 500 and 600 pounds per square inch, between a sixth and tenth of those existing at higher levels.

Now the nature of those geologic actions which oppose isostasy, both the great compressive movements and the great cycles of erosion and sedimentation, are such that they tend to destroy the isostatic adjustments of whole continents and large parts of continents. By these broad actions they tend to bring larger and larger stress-differences upon the zone lying more than 200 km. deep. The limitation of their action as shown by the dominance of regional isostasy is therefore to be regarded as an effect of weakness in that zone. This then is another proof of the reality of an asthenosphere. The proof in Part VI depended upon the dynamics necessary for isostatic undertow; the proof in this part depends upon the limitations of stress with depth as measured by the existing departures from isostasy.

This is as far as the present fragmentary data and imperfect theory can safely go, but in order to visualize the arguments a curve of strength is given which shows how great a falling-off of strength there is from the upper part of the lithosphere to the middle of the asthenosphere. Below, the strength undoubtedly again increases, but the evidence for that is supplied by other lines of research than that opened by the geodetic data.

The results of this part suggest that in future investigations by mathematicians upon the elastic competence of the earth, a probable case would be to consider the isostatic compensation as uniformly tapering out through a depth twice as great as the depth given for uniformly distributed compensation, that is, tapering out through about 244–254 km. Further, it is not in accordance with nature to assume that at this depth all shearing stresses disappear. Such an assumption brings in artificialities nearly as great as those involved in Darwin's assumption of no isostatic compensation. Rather should the stress relations be solved as limited by some such curve as is here shown, and determined by the nature of the departures from isostasy. It is possible that this may add still further difficulties to the mathematical treatment of the subject, yet only by closer recognition of the realities of nature can mathematical analysis become of increasing value.

SECTION A, PRESENTATION OF THEORY

RELATIONS OF LOADS AND STRESSES

Stresses imposed by harmonic surface loads.—A harmonic series gives a succession of sweeping curves such as are shown in Fig. 15. The vertical scale may be made of any size and the curves may be regarded as sections across a series of hills and valleys, or, on progressively larger scales, anticlinoria and synclinoria, geanticlines and geosynclines, continents and ocean basins. The parts of the curves convex upward will then represent loads above the mean surface and give rise to stresses acting downward. The broad hollows give negative loads and the surface beneath is strained upward by the pressures from surrounding regions. The inequalities of the earth's surface may be taken as approximating to harmonic undulations of simple or complex nature. By so taking them, the stresses which they produce on the earth's interior may be evaluated.

G. H. Darwin treated this problem in his paper "On the Stresses Caused in the Interior of the Earth by the Weight of Continents and Mountains."[1] In this are investigated the stresses given by positive and negative loads whose distribution follows a law

[1] *Phil. Trans. Royal Soc.*, CLXXIII (1882), 187–230.

of zonal harmonics arranged on the surface of a sphere. The harmonic series of the second order corresponds to oblateness of a spheroid and also serves as a basis for computing the tidal strains. The zonal harmonics of the fourth order correspond to an equatorial continent and two polar continents, the eighth order adds to these two annular continents in about latitude 45°, and so on. The higher orders, above thirty, correspond to a succession of anticlinoria and synclinoria, or mountains and valleys. For all above the second harmonic the depth of maximum stress lies within the outer half of the earth's radius.

Darwin's solutions were made on the assumption that there are no differences of density beneath continents and oceans and that all the relief of the earth is upheld by its rigidity. He reached the conclusion that continents such as Africa and America gave a maximum stress-difference of about four tons per square inch at a depth of about 1,100 miles. The later demonstration of the existence of regional isostasy nullifies this conclusion except for the amount by which the topography of large areas is not completely compensated. Even this part can to some extent be regarded as sustained by a rigid crust floating upon a deeper zone which acts dynamically nearly as a fluid. There are reasons for believing, however, that this latter conception is extreme in the other direction and not justified by the evidence. It is thought that by the collective support of the arguments brought out in this part the assumption will be finally justified—that for the outstanding loads not in isostatic equilibrium the work of Darwin continues to apply.

In the mathematical analysis, the loads which represent the areas and heights of the regional departures from isostasy are regarded as members of an infinite harmonic series of ridges and furrows disposed on a plane. One wave-length is the distance from crest to crest, or mid-furrow to mid-furrow. Darwin showed, as illustrated in Figs. 15 and 16, that the magnitude of the stress-difference at any point within the crust due to a surficial harmonic load depended upon the depth below the mean horizontal surface measured in terms of the wave-length and not at all on the position of the point considered with reference to the ridges and furrows. Further in regard to the direction of the stress-difference, it is

shown, as illustrated in Fig. 15, that in passing uniformly and horizontally through the crust on a line at right angles to the direction of the ridges, the stress axes revolve with a uniform angular velocity. In relation to depth, the maximum stress-difference, as shown in Figs. 15 and 16, occurs at a depth equal to $\frac{1}{2\pi}$ of the wave-length and is then equal to $2\, gwh\epsilon^{-1}$ or in gravitation units of force to $0.736wh$, in which h is the height from the mean plane to the top or bottom of the undulations and w is the weight of a unit volume.

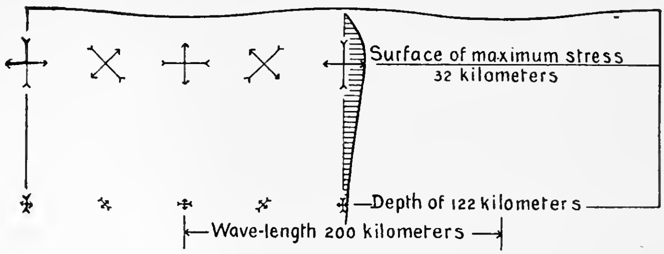

FIG. 15.—Diagram showing in vertical section uncompensated harmonic mountains and valleys with relative magnitude and direction of stress-differences, which they impose on the crust below. Mountain crests drawn as 5 km. above valley bottoms. Wave-length 200 km. Stresses shown to a depth of 122 km. Maximum stress for this wave-length is at 32 km.

It is important to note that the value of this maximum depends only on the height and density of the mountains and is independent of the distance from crest to crest. The depth at which this maximum is reached depends, on the other hand, upon the wave-length and not upon the height or density of the mountains. The effect of a doubling of the wave-length upon the vertical distribution of the stress-differences is shown in Fig. 16.

It is seen that the lateral pressure due to the elevations, instead of being at the surface as it would be under hydrostatic conditions or as in completely compensated mountains and valleys with the special distribution of density assumed by Love,[1] is at a depth of about one-sixth of the wave-length. The maximum stress is, furthermore, but 37 per cent of the full amount of the hydrostatic

[1] *Some Problems of Geodynamics* (1911), chaps. ii and iii.

lateral pressure. Fig. 16 shows that uncompensated features with a wave-length up to 200 km. impose the stresses almost wholly within the lithosphere, taking this as limited by a depth of 122 km.

The crust increases in strength to a certain maximum, perhaps from 10 to 30 km. deep, as shown by the experiments of Adams.

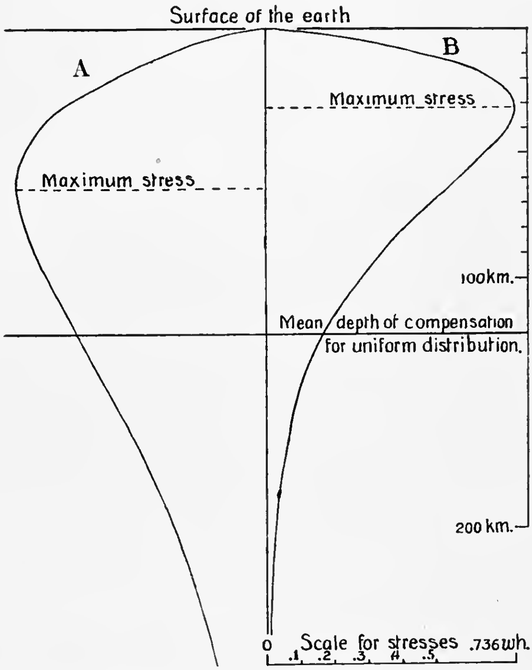

FIG. 16.—Diagram showing the distribution in the crust of stress-differences due to parallel uncompensated harmonic mountains and valleys: A, curve showing relative magnitude and depth of stress-differences corresponding to a distance of 400 km. between crests of parallel mountain ranges. B, curve showing the same for a distance of 200 km. between crests of parallel mountain ranges.

At greater depth the strength, according to the theory developed in Part VI, and as recognized by Love and others in treating of isostasy, is to be regarded as decreasing and passing by transition into the asthenosphere. Consequently it is seen that the distribution of stress imposed by a wave-length of 200 km. conforms well to the distribution of strength, the greatest strain coming on the strongest part.

Darwin obtained his results by making the initial assumption that the earth substance was incompressible but possessed elasticity of form. The introduction of the factor of compressibility Darwin showed to affect the result largely in the case of the second harmonic, but for harmonics of infinitely high orders the resulting stresses are independent of the modulus of compression. Consequently he states, "it may be concluded that except for the lower harmonic inequalities compressibility introduces but little change in our results."[1]

Modifications imposed by long and large wave-lengths.—In Fig. 16, curve A shows that for a wave-length of 400 km. the depth of maximum stress is 64 km. and at 122 km. the stress is 75 per cent of the maximum. If this should be regarded as the beginning of the asthenosphere it would mean that a large part of the stress would be thrown on to the zone incapable of sustaining large stress-differences. If the wave-length became 2,000 km. the lithosphere would be subjected to but small stress-differences and the maximum strain would occur at a depth of 320 km., the middle of the sphere of weakness. In order that Darwin's solution should hold for these cases the height of the arches would have to be so small that the resulting stress-differences would not exceed the elastic limit of the asthenosphere. For greater loads disposed on the surface in these large wave-lengths the stress relations would approach those of a rigid crust overlying a fluid substratum. Of this problem Darwin states, "The evaluation of stresses in a crust, with fluid beneath, would be tedious, but not more difficult than the present investigation" (on the stresses caused by the weight of continents and mountains).[2] It is a different problem from that solved by Love; the latter considering the stresses in such a crust caused by a condition of isostasy, not by a lack of isostasy. The limited mathematical training of the present writer does not permit here the definite solution of this problem, but some general observations can be made.

For wave-lengths very large in comparison with the depth of the lithosphere the stress-differences, if confined within the crust, approach those existing in a continuous beam, each span being

[1] *Scientific Papers*, II, 500. [2] *Ibid.*, II, 502, footnote.

stressed by a continuous load, greatest at the center in accordance with the harmonic curve and acting in the reverse direction from the adjacent spans. This is a very simple limiting case. The maximum bending moment would be on the cross-section at the crest of each downward and upward arch. For a given height of load the bending moment would increase with the square of the span or wave-length. The maximum bending stresses would be horizontal, acting as tensile and compressive stresses at the top and bottom of the lithosphere. In the middle of the lithosphere there would exist a neutral surface suffering neither tension nor compression, but subjected to horizontal shear. The theory of beams shows that the strength is limited by the marginal tensions and compressions, not by the internal shear. As the lithosphere is, however, weakest on the upper and lower margins, its material is poorly arranged to resist the bending stresses. The greatest resistance to bending in a certain plane is given by the form of an I-beam, but the crust is analogous to a beam in which a single flange should intersect its middle, giving a cross-shaped section. The earth's crust is consequently a peculiarly weak structure to resist harmonic loads of great wave-length, and as the strength varies inversely with the bending moment it varies inversely with the square of the wave-length. It is seen then that wave-lengths of continental breadth are very poorly supported by the strength of the crust, but if they reach notable amplitude must rest chiefly upon the asthenosphere. The consideration of the stress diagram given by a wave-length of 200 km. showed why very pronounced departures from isostasy can occur in one direction over areas up to at least 100 km. across and why marked regional compensation extends commonly to limits of 100–200 km. radius. The stress effects produced by harmonic loads a thousand kilometers or more in a wave-length show, on the other hand, why regional compensation of the same vertical magnitude cannot extend effectively across a whole continent. It explains why the United States as a whole is in nearly perfect isostatic equilibrium with respect to the ocean basins.

Nature of stresses imposed by internal loads.—Take the case of harmonic loads distributed on a plane *S–S*, Fig. 17, within an

indefinitely extended elastic solid. The amount and direction of the vertical stress upon this plane are shown by the vertical lines, the scale of stresses being one-twentieth of the value for the stress-diagram shown by the horizontal lines. On this plane, *S–S*, a small unit mass is subjected to stress equal in all directions and not to stress-difference, since the stress is essentially the same on the small contiguous unit masses. The reasoning is the same as

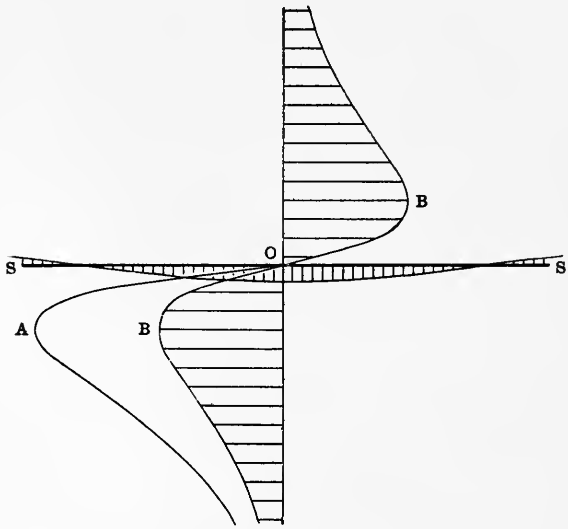

FIG. 17.—Stress-diagrams for harmonic loads distributed on a plane: A, diagram for loads upon a limiting surface of a solid extending indefinitely from this surface. B, diagram for loads upon a plane within an indefinitely extended solid. The scale for loads is one-twentieth of the scale for the resulting stresses. A little more than one-half wave-length is shown.

that for harmonic loads distributed on the limiting surface of a solid except that here there is an indefinitely extended solid on each side of the plane. The stress at any point of the plane acts positively on one side, negatively on the other. Half of the load will be carried on each side. Consequently if *OA* is the curve showing the stress-differences at various depths for a harmonic load on the surface of the earth, then *BOB* will be the stress-curve for the same load carried on a plane deep within the lithosphere. For this to be approximately true, however, the wave-length would have to be

small in comparison with the depth of the loaded surface. Then the upper half of the stress-diagram will lie within the lithosphere. The lower half of the curve would also have to lie within the elastic competence of the crust for corresponding depths. Suppose the plane to lie near to either boundary of the lithosphere. The case now approaches that of a surface load, one side of the stress-diagram becomes largely cut off and the other increases. The exact analysis is of course difficult and will not be attempted.

Assume the loaded plane to be at a depth of 61 km., half the depth of the lithosphere. The strength of the middle would not then be utilized for support of the load. For a wave-length up to 100 km. Fig. 17 shows fairly well the distribution of stress and it would be contained mostly within the middle of the lithosphere. For wave-lengths of 200 km., however, the margins of the lithosphere would be subjected to greater strain than the interior, the stress-diagram would be modified toward that existing in a loaded beam, and, if the margins are weak, the structure is poorly adapted to support the load. If the loaded plane is at greater depth, the same wave-length will throw a greater proportion of the strain upon the asthenosphere and the deeper parts of the lithosphere. If these are incapable of supporting the resulting stresses, again a modification of the diagram would occur, involving bending moments in the stronger part of the crust.

If, now, it be assumed that the upper half of the lithosphere is decidedly stronger than the lower half and that the maximum strength is at some depth below the surface, it is seen that the maximum outstanding masses which the crust could carry would be disposed in the outer quarter or third of the lithosphere. But this is just what was found to be the case as the result of the studies made in Part V. Therefore the accordance between the geodetic evidence and the consequences of the assumption raise it to the dignity of a presumption. It may be taken as a working hypothesis that the greater outstanding masses are limited in their positions by the limitations of crustal strength. Mental reservation must be made, however, as to the possibility of other more important determining factors, such for example as the nature of igneous activity, in limiting the zone of large outstanding masses. An

accordance of fact with theory is not a proof, but it raises a presumption that the theory is correct.

Nature of stresses imposed by perfect isostasy.—This topic, although not directly in line with the subject of this chapter, must receive brief mention here since the stresses in the crust are compounded of those due to the departures from isostasy with those resulting from a state of perfect isostasy. The stress resulting from the isostatic support of continents and mountains has been ably worked out by Love.[1] But his treatment started with the limiting though improbable assumption that at a depth of one-fiftieth of the earth's radius all stress-differences disappeared, as though the layer below were truly fluid. This required a complicated and equally improbable curve of density, opposite in sign above and below, in order that the topography should be compensated, the ocean surface remain a level surface, and yet the stress-differences become zero at the required depth. Nevertheless the solution is valuable as a limiting case in showing the general character of the internal stresses which must exist. He showed that isostatically compensated harmonic mountains and valleys gave maximum stress-differences on the axial lines of mountains and valleys and that it amounted to about one-fourth of the theoretical hydrostatic pressure. The stresses decrease rapidly with depth.

In contrast to Love's hypothesis of the distribution of density, that of Hayford may be considered. This is that the excess or defect in density needed for compensation is uniformly distributed to a depth of 122 km. Again, a rigorous mathematical treatment must be left to those competent to undertake it, but it would appear that such distributions of density would throw very considerable stress-differences within the asthenosphere; or, if this was incompetent to carry such, would bring large stress-differences upon the bottom of the zone of compensation, opposite in sign to that in the upper half, whereas at an intermediate level depending upon the wave-length would be a region of no stress-difference. This distribution of stress resulting from the hypothesis may be taken as a strong argument against the existence of a uniform distribution

[1] *Some Problems of Geodynamics* (1911).

to the isostatic compensation. In order to have the larger relief and its compensation fitted to a crust strongest in its upper part and shading into a zone of weakness, the zone of compensation should die out with depth and lie mostly in the upper half of the zone of strength, since the stress-differences would die out at a depth greater than the disappearance of the compensation. This conclusion is seen to be in closer accord with several other lines of evidence which have been noted than is the contrary assumption of uniform compensation.

[*To be continued*]

THE STRENGTH OF THE EARTH'S CRUST

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PART VII. VARIATIONS OF STRENGTH WITH DEPTH AS SHOWN BY THE NATURE OF DEPARTURES FROM ISOSTASY

SECTION B, APPLICATIONS OF THE THEORY

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GEODETIC EVIDENCE AS TO LIMITING HEIGHTS AND WAVE-LENGTHS

Measurements of strength by maximum loads.—In the course of geologic time the internal forces of igneous intrusion and tangential compression, the external forces of erosion and sedimentation, have tended to strain the crust to its limits of strength, and the degree of isostasy which exists constitutes a measure of those limits. Small loads of large wave-length and large loads of small wave-length will tend to rise to maxima which may be used in connection with the theory of distribution of stress, as considered in Part VII, Section A, to give an approximate idea of the limits and distribution of strength in the crust.

The problem is to find the maximum vertical load and its relation to wave-length acting over areas which may be regarded as

forming roughly a harmonic series. The theory applies best where elongated unit areas are flanked by areas of opposite sign. But even where a single axis of uncompensated elevation or depression is surrounded with a region of mean elevation it may be regarded as a half of a wave sustained by the rigidity of the earth. The stresses in the vertical plane through its crest line would appear to be less than, but not so greatly different from, what they would be for a continued series. Where the load is of oval instead of zonal distribution the stresses would also be somewhat diminished in case the oval is surrounded by a neutral region, but if a series of ovals of opposite signs is analogous to two intersecting wave-series, although the stresses would be complicated and are not in general the sum of the stresses due to the separate series, yet it does not appear that the extreme maxima would be necessarily less than the sum, and many of the maxima would be greater than the maxima of the separate series.

Harmonic loads with short wave-lengths.—Horizontal compression builds mountain folds of which the individual ranges are clearly the results of compression and not of isostatic elevation. Erosion dissects an elevated country on a pattern of a certain scale, deep valleys of erosion separating the crust remnants above base-level which are not yet consumed. These actions produce a surface relief which corresponds roughly to various harmonic series of appreciable amplitude and wave-length, but in this section will be considered only the geodetic evidence of variable mass not isostatically compensated, much of the variation being due to the concealed heterogeneities of density.

The distance between Washington, D.C., and Hoboken, New Jersey, as estimated in Part V, Section B, is 326 km. Within this distance are three intermediate stations and the two limiting stations, each station showing a gravity anomaly opposite in sign to that of the adjacent stations. There must be then at least two wave-lengths. The average change of anomaly between the adjacent stations is 0.021 dyne. As it is wholly improbable that the stations are located at the crest lines of the waves, the whole amplitude may safely be taken as at least 0.026 and the wave-length 160 km.

Seattle and Olympia are 80 km. apart and the difference of their anomalies is 0.126 dyne. If the anomalies had been measured at the points giving maximum values they would certainly give a difference of at least 0.130 or perhaps 0.140, about five times the amplitude of the variations between Washington and Hoboken. This large value seems, however, to be exceptional for the United States and may constitute but a single wave. We may take it, however, as showing that the crust can sustain a harmonic load of 160 km. (100 miles) wave-length and total amplitude of 0.120 dyne. For the reasons explained in Part IV, especially on p. 304, the divisor to be used to turn this anomaly into the equivalence of rock measured in feet could not be over 0.0018 and a better figure for the interpretation of this short wave-length is 0.0015. This gives an amplitude of 8,000 feet (2,440 m.). The part below the mean level is then 50 miles wide and 4,000 feet deep, the adjacent positive parts of the wave being of equal dimensions but in the opposite direction. This is of the same order of relief as the larger ranges of folded mountains and intermontane valleys. The stresses which this harmonic series imposes upon the crust are shown by curve A of Fig. 18.

Maximum loads for mean wave-lengths.—In Part II it was argued that the evidence of anomalies from mountain stations showed regional compensation on the average probably to the outer limit of zone O, radius 166.7 km., diameter consequently 333.4 km.

Over the United States in general the intercepts of the areas of grouped deflections averaged 180 miles. The average diameter would therefore doubtless be as great as 200 miles (320 km.).

On Bowie's map of the New Method gravity anomalies,[1] it is seen that the distances from pronounced maxima to pronounced minima average 250–350 km.

From these several lines of evidence we may conclude with some confidence that the half wave-length for pronounced anomalies in the United States averages near 300–350 km. The wave-length is therefore 600–700 km. (373–435 miles). A wave-length of 600 km. will be taken. The pronounced maxima for these waves runs from plus or minus 0.030 to 0.060 dyne. The real maxima

[1] This article, Part II, *Jour. Geol.*, XXII (1914), 153.

are to be regarded in most cases as situated to one side of the stations and somewhat greater. But exceptional and local maxima must be smoothed out to form a part of the harmonic curve. It is, furthermore, the difference of anomaly between adjacent opposite phases which is the significant feature. This difference runs from 0.060 to 0.080. The latter figure will be chosen. For this wave-length, representing a certain unit area of attraction, the best divisor is perhaps to take 0.0024 dyne of anomaly as equivalent to 100 feet of rock. An anomaly of 0.080 dyne is on that basis equivalent to 3,330 feet (1,015 m.) of rock. The crust of the United States sustains, therefore, harmonic loads 600 km. (373 miles) in wave-length and 1,015 m. (3,330 feet) in total amplitude. The stresses which this wave-series imposes on the crust are shown by curve B, Fig. 18.

Departures from isostasy of large wave-lengths.—For the continent as a whole and in its relations to the ocean basins isostasy is nearly perfect; but the question rises here, how nearly? The first term of the gravity formula for the Vienna system of gravity observations is 978.046 dynes. The first term for the Potsdam system is 978.030. The first term for the United States system after rejecting the Seattle anomalies is, as shown by Bowie, 978.038 dynes. These respective systems differ as a whole by these amounts. We have no right to assume that any one is absolutely correct. The whole of the United States system may lie a little above or below the level giving isostatic compensation with respect to the average surrounding ocean basins, or with respect to the entire earth. The mean value for the United States suggests, however, that, as a whole, the continent lies within a few hundred feet, possibly less than one hundred feet, of the level which would give perfect isostatic equilibrium.

Let us consider next its larger parts. These can be compared with each other and with the United States as a whole. Although, as discussed in Part IV, the map of gravity anomalies lacks detail, the grouping of many stations of like sign into large areas gives confidence in the conclusion that there are regional departures from isostasy. These are of two or three orders of magnitude, of which the areally smaller have been discussed. To bring out the

areally larger we must draw boundaries about large regions which show a dominance of anomalies of one sign. These boundaries, however, must be taken so as to give compact unit areas, so as not to obtain an unreal result by the political expedient of gerrymandering the districts.

Select as a center the point whose geographic co-ordinates are lat. 42°, long. 102°. Describe about this center a circle of 850 km. radius. This includes an area equal to 29 per cent of the area of the United States. It should be taken as including the negative anomaly station 99 on its southern border. This circle covers a large positive region which could be made still more positive by an extension of its boundaries to the northeast over Wisconsin and Michigan. Within this circle are distributed with a fair degree of uniformity 31 of the 122 gravity stations of the United States. The mean with regard to sign of the anomalies of these 31 stations referred to the United States mean with regard to sign is +0.010 dyne. As the mean without regard to sign of all stations in the United States excluding Seattle is only 0.018, it is seen that this positive region stands out clearly from the general average.

West of this circle and, on the south, to the west of long. 107° there are 21 stations, including one of the two Seattle stations. These mark a broad region of negative anomaly. The mean anomaly with regard to sign is −0.017 dyne. There seems to be no reason for completely omitting the exceptionally large Seattle anomalies. One of them has therefore been retained, but if both are omitted the mean is still −0.013. The value of −0.017 will here be adopted. The difference of the means of the central and western regions is consequently 0.027 dyne. Let these be regarded as the positive and negative phases of an harmonic wave and the mean departure of the two phases becomes 0.0135 from each side of the mean plane. Now it may be computed for a harmonic wave represented by the formula $y=A \sin Bx$ that the mean height of the wave above the mid-plane is 64 per cent of the crest height. From mid-plane to crest of this wave-series is therefore 0.021. From the large negative anomalies along the Pacific coast it would seem that this negative zone must extend somewhat further. The wave-length of this series is consequently between

2,600 and 3,000 km. A mean value of 2,800 km. (1,740 miles) will be chosen. From the breadth of half a wave-length it appears that 0.0034 dyne of anomaly may be taken as equivalent to 100 feet of rock. This gives the crest and trough as 625 feet (190 m.) from the mean plane, a total amplitude of 1,250 feet (380 m.). The stress-differences which this wave-series throws upon an earth elastically competent throughout to bear the stresses are shown by curve C, Fig. 18. Helmert has published an extensive paper dealing with the force of gravity and the distribution of mass in the crust of the earth,[1] to which the writer's attention has been called recently by Professor Pierpont, of the mathematical department of Yale University. In this paper Helmert adopts the hypothesis of regional isostasy and finds his results confirmatory of it, but not in accord with the hypothesis of close and local isostatic adjustment. His work is especially valuable as confirmatory of the present conclusions, since it deals with regions outside of the United States. As he does not, however, compute the corrections due to the distant large irregularities of topography, his figures cannot be directly compared with Hayford's New Method anomalies. Nevertheless his conclusions as to the existence of broad regional excesses or defects of mass are comparable to those here reached. Under the section on the horizontal displacement of compensation and extended excesses and defects of mass[2] he sums up part of the evidence in the following statement: "We have then to deal with a continuous region of positive total gravity disturbance in Europe 1,000 km. broad and also with a region of negative disturbance in Asia of at least 500 km. breadth, both possessing great linear extension."

RELATIONS OF ACTUAL STRESSES TO THE SUM OF HARMONIC WAVES

Both Darwin and Love point out that the actual stress-differences imposed by the superposition of different harmonic waves is not in general the sum of the individual stress-differences. Darwin, however, states the special conditions under which the

[1] "Die Schwerkraft und die Massenverteilung der Erde," *Ency. Math. Wissenschaft*, Band VI, 1, B, Heft 2 (1910), pp. 85–177.

[2] *Op. cit.*, pp. 152–54.

resultant is the sum of the individual stress-differences.[1] The three waves which have been considered are types which coexist and are superimposed. The total stress which they give would vary from

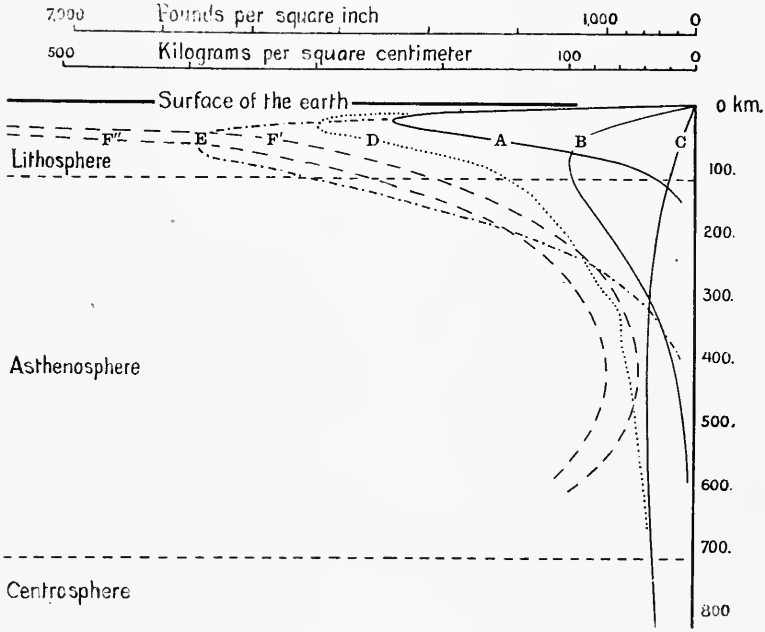

FIG. 18.—Stress-curves for harmonic waves on an earth elastically competent throughout, the waves representing departures from isostasy in the United States as given by analyzing the geodetic data into the following harmonic waves:

A, wave-length 160 km., amplitude 0.120 dyne = 2,440 m. of rock.

B, wave-length 600 km., amplitude 0.080 dyne = 1,015 m., of rock.

C, wave-length 2,800 km., amplitude 0.042 dyne = 380 m. of rock.

D is the sum of A, B, and C.

E, wave-length 400 km., amplitude 0.366 dyne = 4,000 m. of rock (from the Pacific Ocean).

F′, curve of strength suggested by geodetic evidence from the United States.

F″, curve of strength suggested by geologic evidence from various parts of the world.

complete neutralization up to their sum as a possible maximum. Curve D represents such an addition of A, B, and C, Fig. 18. There are reasons why this curve may be taken as a fairer representation of the maximum stress conditions under the United States than

[1] *Scientific Papers*, II, 492.

any individual curve, even though there may be no place where the culminating phases of like sign all coincide and become additive. These reasons are found in the subsurface location of those loads due to outstanding density and also to the added stresses due to isostatic compensation. These causes result in throwing a greater stress upon the outer parts of the lithosphere and also serve to broaden downward the stress diagrams. Furthermore, the stresses due to isostatic compensation of continents would appear to be in reality much greater under the margins than the small values computed by Love,[1] since he has taken the continents as having the broad sweeping surfaces of a harmonic nature, whereas, as a matter of fact, the continents slope off rather abruptly to the depths of the oceans. Facing the Pacific in fact, the two Americas show high mountain elevations. This would cause the stresses in the vicinity of these continental margins to resemble those imposed by a great mountain chain and its isostatic compensation rather than those imposed by the breadth of a continent. If isostatic compensation is complete under mountain slopes, Love shows for the cases computed by him that the maximum stress is about equal to that given by a column of rock one-fourth the total height from mountain crest to valley bottom. If the abyssal slopes of the continental platforms be taken as averaging 3–4 km. in elevation and 50–100 km. in width, it is seen that, even if fully compensated, they add stresses to the crust which may approach in magnitude one-half of the stresses shown by curve A of Fig. 18. The extreme depths of slope are much greater and it is clear that isostatic compensation cannot be exact under these great reliefs. Therefore we may conclude that curve D does not overestimate the maximum stresses imposed by the irregularities of the crust, both compensated and uncompensated, as indicated by geodetic evidence within the United States and especially along its ocean borders. This investigation, however, has been of a general nature and is designed merely to establish an order of magnitude. It remains for future work to make more precise analyses for each locality from the data which may be acquired, and especially to investigate quantitatively the problems offered by critical areas.

[1] Some Problems of Geodynamics (1911), chap. ii.

GEOLOGIC SUGGESTIONS AS TO MAGNITUDES OF CRUSTAL STRESSES

Submarine geanticlines and geosynclines.—Here will be considered some geologic illustrations of departures from isostasy, arranged in order of harmonic wave-length. They are to be compared with the results obtained from the study of the geodetic data. Most of the geologic evidence is merely suggestive, not conclusive, since diametrically opposite opinions are held as to the probability of the visible load being offset by an invisible compensation. As suggestions, however, they are none the less valuable, and point the way to needed geodetic observations.

The mountain folds advance from Asia over the floor of the Pacific Ocean, forming the system named by Suess the Oceanides. Mostly hidden beneath the ocean surface, they have been but little affected by erosion. Their ridges and deeps mark the greatest mountain reliefs of the globe. It is probable that here, if anywhere, tangential pressures have forced the crust into folds whose height combined with span is as great as the strength of the crust can endure. To what degree the elevations and depressions are compensated is, however, unknown, and the great arches are supported in part by the lateral pressure of the ocean water. It is quite possible if not probable that appreciable changes of deep-seated density may accompany the growth of such ridges, especially as they mostly exhibit a volcanic activity and are to a greater or less extent structures built up by igneous extrusion. It is not at all probable, however, that they are completely or possibly even largely compensated, but where the mountain folds and trough-like deeps broaden into plateaus or anti-plateaus the presumption is strengthened that the forms may there be isostatically compensated to a large degree. Such plateaus or anti-plateaus cannot then be used in the present argument. The ridges and troughs, however, show in their forms, as has been stated, modes of construction which are not conditioned on isostasy. Let attention be turned then to the folds of the ocean floor.

Passing from west to east, first may be noted between the Philippines, Borneo, and New Guinea a complex of ridges and basin-like deeps. The larger wave-length of that region runs from 300 to 500 km. The Ladrone Islands and Nero Deep give a

distance of about 150 km. from crest to trough and a wave-length of 400–500 km., these folds and many others exhibiting a lack of symmetry. The existence of strong folding pressures and a tendency to overthrusting and secondary vertical faulting seem to be expressed therein. The great fold passing north from New Zealand and showing as the Kermadec and Tonga islands with their fore-deeps gives a distance from crest to trough of 120–180 km., a wave-length of 400–500 km. Lower California and the troughs on each side show a wave-length of 350 km. The region of the Lesser Antilles is tectonically a northward branching of the Andean mountain system and shows, like the folds of the Pacific, crustal undulations with a wave-length of 350 km. We may conclude then that these folds of the ocean floor have a marked tendency to a wave-length of 300–500 km., there being commonly one great asymmetric fold passing out into subordinate marginal folds. The volcanic chain of the Hawaiian Islands shows, however, no related deeps and has a half wave-length of about 200 km.

Hayford and Bowie have given the New Method anomalies for a few stations in these regions.[1] A portion of the data has been abstracted and given in Part IV of the present article.[2] Four observations of Hecker's for the Tonga Plateau and Tonga Deep are given. They may not be of high value, since the method has been criticized as not possessing accuracy comparable to observations made by pendulum upon land. Furthermore, the four observations, two over the plateau and two over the deep, are spread through a distance of 5,100 km. along the axis of the structure instead of being taken on a transverse section. Nevertheless, as the reliefs and the corresponding anomalies are all of great magnitude, the errors become relatively small and the mean of the observations is therefore of some value. The two New Method anomalies for the Tonga Plateau give a mean of $+0.202$ dyne, the depth of water being 2,700 m. The two New Method anomalies for the Tonga Deep give a mean of -0.172 dyne, the depth of water averaging 7,500 m. If the amplitude

[1] *The Effect of Topography and Isostatic Compensation upon the Intensity of Gravity*, 1912, p. 81.

[2] *Jour. Geol.*, XXII (1914), 311.

of the uncompensated portion of the crust-waves be measured in terms of anomaly by taking the algebraic sum of the anomalies over the plateau and the deep, a total amplitude is obtained of 0.374 or a half-amplitude of 0.187. On the island of Hawaii an observation on Mauna Kea at an elevation of 3,981 m. gave a New Method anomaly of +0.183, almost the same figure as the half-amplitude for the great Tonga crust-wave.

Helmert has discussed the gravity disturbances found in the Hawaiian Islands and states of them: "For the Hawaiian Islands it must be concluded on the whole that a part of the mass gives rise to positive gravity disturbances and only the remainder is isostatically supported. If the disturbances were produced solely by the mass of the islands the values of Δg and $\Delta g''$ [the disturbances of gravity] would be somewhat greater than they are found."[1]

From this review of the mountain chains of the Pacific it may be concluded that the ocean floor can sustain a harmonic wave-length of 400 km. which gives an anomaly at the crest lines as great as that observed on Mauna Kea, 0.183 dyne. To interpret this as an equivalent load of rock a divisor must be selected. The divisor depends upon the depth and distribution of compensation and the area of the region of outstanding mass. As shown in Part IV, p. 311, 0.0024 dyne might reasonably be chosen as the amount of anomaly equivalent to 100 feet of rock, but for these great loads it is desirable to lean toward the side of an underestimate. Therefore 0.0030 will be taken as the divisor. This gives 1,868 m. as the crest height of the uncompensated portion of the Hawaiian mountain chain. The same applies to the Tonga fold. If, however, 0.0024 should be chosen as the divisor, then 0.183 dyne of anomaly would correspond to a half-amplitude of 2,334 m.

It may be taken then as fairly certain that these great mountain chains show reliefs which depart as much as 2,000 m. above and below the mean level which would give perfect isostasy. It may be concluded in consequence that the oceanic crust can sustain a harmonic wave-length of 400 km. with an uncompensated amplitude measured by 4,000 m. of rock. The diagram of

[1] "Die Schwerkraft und die Massenverteilung der Erde," *Ency. math. Wissenschaften*, Band VI, 1, B, Heft 2, (1910), p. 133.

stress-differences for this is shown in curve E, Fig. 18. But if the rock has a density of 2.67 and the sea-water a density of 1.03, this corresponds to an amplitude beneath the ocean surface of 6,513 m. of uncompensated rock. This is only about two-thirds of the maximum relief which is observed, so that it is well within the limits of possibility. These few available figures suggest that the sharp submarine ridges and deeps may not be more than one-third or two-thirds compensated.

The Niger delta.—Reverting to the discussion of the Niger delta given in Part I, it is seen that there is no evidence of depression around its margin. It may be taken then as the positive half of a harmonic wave well within the limits of crustal strength. If the section of the delta be taken as given in Figs. 3 and 4, pp. 31, 43, it is seen that the load is disk-like in form, instead of being indefinitely elongated at right angles to the section in accordance with the form of a zonal harmonic. It seems likely, because of these two departures from the nature of a harmonic series, that the stresses beneath it are not more than half of those which would be given by the completed harmonic curve. As it is merely the order of magnitude of the stress-differences which we may hope to attain we may proceed in accordance with these rough assumptions. It is seen that the section of the delta shows a half wave-length of about 300 km. and a maximum thickness equivalent to 1,650 m. of rock upon land. This corresponds to the half-amplitude or thickness above the mean plane. If half of this is taken as a measure of the stress, it gives a wave-length of 600 km. and a total amplitude of 1,650 m. The stress-curve for this harmonic series is 60 per cent larger than the stresses due to the outstanding masses of the same wave-length as given by the geodetic evidence in the United States and shown in curve B, Fig. 18. As the estimate from the Niger delta is very imperfect and unchecked by pendulum observations reduced by the New Method, the stress-curve is not plotted.

The existing continental ice sheets.—Two ice sheets of sub-continental proportions remain in existence, the Greenland and Antarctic. They form great plateaus sloping upward from the margins; the Greenland sheet reaching elevations at its center

between 9,000 and 10,000 feet, the Antarctic attaining to about 11,000 feet. The average thickness of the ice must be thousands of feet in each case. The development of these ice caps during the refrigeration of climate which marked the later Tertiary must have imposed upon the crust great loads of wide span. If isostatic equilibrium was previously complete to a large degree, the ice mantles should give valuable measures of crustal strength. For this purpose, however, a set of gravity measurements should be carried inland and reduced by the Hayfordian method. The facts that these two ice-mantled areas are both high plateaus, and that no other adjacent unglaciated land is of similar topographic character, suggest that these regions may be competent to carry great thicknesses of ice without isostatic yielding. There is no present basis, however, for making a quantitative estimate. It must be borne in mind, furthermore, that the ice mantle is only about one-third of the density of rock and that lofty mountains exist in both regions, showing that these lands would possess considerable mean elevation even without the presence of the ice. The effect of the difference of density between ice and rock may be appreciated by considering that an ice sheet 3,000 feet in thickness would possess the same mass as a layer of rock 1,000 feet thick. For isostasy to remain perfect after the development of this ice sheet, the crust would have to sink 1,000 feet, but the surface of the ice would still be 2,000 feet above the former level and give an appearance of load which would not in reality exist.

This is a problem meriting research for several reasons. A knowledge of the load which is sustained by these regions would show to what degree the warpings connected with the extinct Pleistocene ice sheets were mere elastic responses to load, to what degree they marked subcrustal plastic flow working toward isostasy. The results could be applied also to the problem as to how far from isostatic equilibrium a continent might come to lie as a result of continent-wide base-leveling in a period of geologic quiet. It seems not impossible that the stress-curve due to the portion of the glacial load which is elastically sustained would give stress-differences greater at a depth of 300–500 km. than those shown by curve C, Fig. 18. Such an investigation may then

be an essential factor in measuring the maximum strength of the lithosphere and more especially the asthenosphere.

Accordance of geologic with geodetic evidence.—The United States and its bordering ocean bottoms is a region of moderate reliefs as compared to the great folds of the ocean floor or of the continent of Eurasia. The geologic forces of folding and uplift have not worked here with their greatest intensity and the central and eastern half of the continent has been affected by the world-involving Cenozoic diastrophism to only a moderate degree. It is to be expected then that the greatest strains upon the crust, the maximum departures from isostasy, would not be found here. In accordance with this expectation it has been seen that by far the greatest New Method gravity anomalies are found in other regions and associated in most cases with the greater reliefs of the globe. The geologic evidence is in harmony; the amount of uncompensated relief, parallel to the geodetic evidence, is greatest for the lesser wave-lengths; but, throughout, the geologic evidence suggests that the actual burdens which can be borne by the crust, as found in regions of culminating stress, are appreciably greater than those detected by geodetic methods as existing in the region of the United States.

If, in some past ages, as during the Appalachian or Sierran revolutions, strains were generated in this continent as great as those found now in some other regions, it would appear that the slow changes of geologic time, of erosion and crustal readjustment, have partially eased the crust of its load. We may have, then, a variable crustal strength—a maximum strength exhibited during and following the crises of great diastrophism; another, lesser strength, which measures the loads which the crust without failing can bear through all of geologic time.

ADJUSTMENT OF LOADS TO THE DISTRIBUTION OF STRENGTH

It has been seen that the departures from flotational equilibrium may become very notable and are of greatest vertical magnitude for wave-lengths from 100 up to 400 km. The strains generated by these loads, if distributed through an elastic crust, consequently

reach maximum values at depths not exceeding 64 km. Is this because the earth shell below the zone of compensation is strong, but for some unrelated reason free from large stress-differences, or is there an absence of such stress-differences because this shell is too weak to bear them? If the latter is true, then the relations of amplitude to wave-length which have been developed in this chapter offer additional proofs of the reality of the existence of the asthenosphere.

The geologic evidence on the evolution of continental structures and elevations leads to the conclusion that the distribution of stress-differences must be in reality the result of the existence of a zone which cannot carry large distortional strains, as may be seen upon brief consideration.

The internal activities of igneous intrusion and of tangential compression do not in themselves work toward isostatic equilibrium, but merely toward accentuation of relief. Erosion and sedimentation, while tending to destroy this relief, are not agents tending to create, but to destroy, such isostatic relations as have developed. All of these activities work on a continental or subcontinental as well as on an orogenic scale, as seen in the Cenozoic history of the broad Cordilleran province, yet while the orogenic departures from isostasy are vertically very great, the continental departures are very moderate. For the latter there must be then some more narrowly limiting condition. This corresponds to the incapacity of a deep zone, the asthenosphere, to carry large stress-differences and the incapacity of the lithosphere in spite of its greater strength to act effectively after the fashion of a beam for loads of great span. The orogenic structures, on the other hand, give maximum stresses much nearer the surface, in the stronger lithosphere; because of their shorter wave-lengths they do not produce in it bending stress as in a loaded beam and affect comparatively little the deeper-seated asthenosphere.

If, then, it is known from the preceding theoretical considerations that the limits of strength of the lithosphere and asthenosphere determine the limits of the departures from isostasy, the analysis of the nature of these departures may lead in turn to a knowledge of the distribution of strength.

CHARACTER OF THE CURVE OF STRENGTH

In curve F′ of Fig. 18 is shown the nature of the curve of strength as suggested by the geodetic evidence from the United States. In curve F″ is shown the nature of the curve as suggested by the departures from isostasy exhibited by the great mountain axes and possibly by the continental ice sheets. These curves may be taken as showing the value of the elastic limit at various depths for permanent stresses. With varying geologic conditions, especially those connected with rising magmas and their emanations, the curve of strength must vary widely, and furthermore no very close parallelism of strength-curve and stress-curve is to be expected. These curves, therefore, are intended to bring out general relations; they are of qualitative, not quantitative value. The drawing of curve F″ somewhat inside of curve E means that below the point of maximum stress in E, as given for a homogeneous elastic earth, the stress is assumed as somewhat greater than the crust at those levels can sustain. Upon the development of this load plastic flow in these deeper levels would take place maintaining the stress within the strength curve for each level; the crust above would come to act to some extent as a bending plate, the stresses within it would increase, chiefly within the upper and lower portions. This added strain would compensate for the yielding below. For the reasons discussed previously, however, showing the structural weakness of the lithosphere as a beam, this action, it is thought, could not go very far, and, in consequence, the loads on the lithosphere are essentially such as to give stresses contained within it, distributed according to Darwin's law. The preceding deals only with that part of the curve of strength which marks the gradation from lithosphere to centrosphere. The relations of this part to those above and below need still to be considered.

The highest stress found for the loads regarded as harmonic waves was for the great folds on the floor of the Pacific Ocean. These were taken as equivalent to harmonic waves of rock of density 2.67, 400 km. in wave-length, and 4,000 m. in amplitude. But even these folds give a maximum stress of only 393 kg. per sq. cm. (5,590 pounds per square inch), and this at a depth of 64 km. At the surface strong limestone or granite can

sustain a stress-difference of 1,750 kg. per sq. cm. (25,000 pounds per square inch), and selected specimens show ultimate breaking strength approaching 2,800 kg. per sq. cm. For stresses of geologic endurance and in the heterogeneous outer crust it is probable, however, that stress limits should be chosen below 1,750 kg. per sq. cm.

The work of Adams and King has shown that small cavities in granite are not closed when the rock is subjected to the pressure and temperature normally existing in the earth at a depth of 11 miles.[1] The presence of occluded gases acting through great lengths of time, by facilitating recrystallization, might affect this result of laboratory experiments, but the capacity of dry rock to sustain even greater cubic pressures without yielding seems to make safe the conclusion that except in the presence of magmatic emanations the crust at a depth of 11 miles (17.7 km.) is able to bear a stress difference of 100,000 pounds per square inch and is at least four times as strong as rock close to the surface.

At twice this depth, however, the temperatures become such that if it were not for the great pressures even dry rocks would approach a molten condition. The presence of high temperatures and of gases which may act as crystallizers presumably becomes dominant at such depths over the effects of the increasing pressures. We many conclude, therefore, that the maximum strength of the crust in regions free from igneous activity is found at levels above rather than below 40 km. and may lie between 20 and 30 km. deep.

To bring to a focus this discussion a tabulation of ratios of strengths for increasing depths may be given, as derived from the strength curves F′, F″ of Fig. 18, the standard being taken as the strength of surface rocks. By giving them merely as ratios and stating that the average strength of the solid rocks at the surface is itself an uncertain quantity owing to complications of structure and composition, the appearance of an undue certainty is avoided.

The general conclusion which stands out from this tabulation is that the weakest part of the asthenosphere is of the order of one one-hundredth of the maximum strength of the lithosphere and is perhaps only a twenty-fifth of that of massive surface rocks. Its

[1] *Jour. Geol.*, XX (1912), 97–138.

limit of capacity for sustaining stress-differences is apparently of the order of 1,000 pounds per square inch, though its weakness may be masked to some extent by the strength above. From the evidence, however, it seems capable of carrying stresses of more than 100 pounds per square inch, but is clearly incapable of carrying stresses of as much as 5,000 pounds per square inch. To reach a

TABLE XXX

Estimated Approximate Ratios Giving the Variation of Strength with Depth as Shown by the Nature of Departures from Isostasy

LITHOSPHERE

Depth in Kilometers	Strength in Percentage
0	100
20	400
25	500
30	400
50	25
100	17

ASTHENOSPHERE

Depth in Kilometers	Strength in Percentage
200	8
300	5
400	4

more definite conclusion the subject must be tested from many angles and is a problem for the geophysicist rather than for the geologist, but the results are of geological importance and the geologic and geodetic data may turn out to have more determinative value on the distribution of strength than the evidence from tides and earthquakes.

[*To be continued*]

THE STRENGTH OF THE EARTH'S CRUST

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PART VIII. PHYSICAL CONDITIONS CONTROLLING THE NATURE OF LITHOSPHERE AND ASTHENOSPHERE

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INTRODUCTION AND SUMMARY

The experiments of F. D. Adams have demonstrated that under combined pressures and temperatures equal to those existing at a depth of eleven miles granite is about seven times stronger than at the surface, showing that, up to at least certain limits, the strength of the crust increases downward. The measurements of tidal deformation of the earth and of the variations of latitude concur furthermore, in proving that the rigidity of the earth as a whole is greater than that of steel. The transmission of earthquake vibrations of a transverse nature through the earth shows, not only that the earth is solid and rigid throughout, but that, on the whole, rigidity increases with depth. In none of these lines of investigation is there any clear suggestion of the existence of a thick shell of weakness—an asthenosphere.

On the other hand, the conclusion that broad areas of the crust rest in approximate isostatic equilibrium seems to imply the

[1] Section B of Part VIII, on "Relations with Other Fields of Geophysics," will be published in the succeeding number of this *Journal*.

existence of a subcrustal zone with but little strength, readily yielding under vertical loads when these are of such breadth that the strains resulting from them are not confined and absorbed within the strong outer crust.

There has thus been developed a paradox, an apparent conflict of evidence which becomes more insistent of explanation with continued accumulation of proofs of high rigidity from the domain of geophysics and of proofs of regional isostasy from the equally precise field of geodesy.

In the consideration of such broad problems, the hope of an ultimate definitive solution rests upon the use of the method of multiple working hypotheses. The surety, significance, and breadth of application of the facts must be established. By these the various hypotheses must be tested and molded. All hypotheses must be kept for further consideration provided they are not positively excluded. In the complexity of relationships there is commonly a complexity of cause, and hypotheses which seem at first to be mutually exclusive may be found to co-operate in giving a completer explanation. Those which originally appeared antagonistic may thus come to be seen as participating and dividing the field of cause between them. A paradox often points to this kind of a conclusion.

To pass to the particular problem of the relations of isostasy to the physical conditions of the earth's interior; the hypothesis developed in this study—of the existence of a zone of weakness underlying a zone of strength—must not be regarded at present as the only available hypothesis. Searching investigation must be carried forward to see if other and possibly antagonistic hypotheses cannot be developed which will equally well co-ordinate and explain the facts. Even if true in the main, it is likely, as has been the case with other hypotheses, that time will show that in certain directions it has been carried too far. Such testing, however, can best be done by others, and after the implications of this hypothesis are seen. In this part, the concluding article of this series, a discussion had best be given of the lines of adjustment by which the hypothesis here favored may be brought into harmony with other fields of geophysical evidence. With this understanding of the relation of

the present investigation to the method of multiple working hypotheses, examination will be made of the paradox which has been drawn between certain conceptions from other lines of investigation and those drawn from this study of crustal strength.

Having given this introductory presentation on what is conceived to be a judicial point of view, we may turn to a review of the conclusions reached in this article. It is pointed out that rigidity is strictly a measure of stiffness; whereas a very different quality, the limit of elastic yielding, or the beginning of flow, is the measure of strength. But mass flowage may take place in a number of quite different ways, according to the nature of the solid and the environing physical and chemical conditions. The elastic limit and hence the strength will differ in the same solid according to the mode of yielding. Four modes may be here enumerated in what is thought to be their order of increasing importance, the fourth mode being that which is conceived as operative especially in the asthenosphere, and serving to maintain the condition of approximate regional isostasy.

First, flowage may take place rapidly by true plastic or molecular flow, as with lead or white-hot iron, the solid, when stressed well beyond the elastic limit, behaving like a viscous fluid. It is not thought that the terrestrial deformations are often carried on with a rapidity which requires true plastic yielding. In fact, under such rapid stresses as those produced by earthquakes and tides it is not improbable that the strength of the earth may progressively increase with depth.

As a second and quite different mode, deformation may take place by molar as distinct from molecular shear. In the zone of fracture this is manifested in jointing and faulting and is emphasized as distinct from rock flowage, but where the fracturing becomes so closely spaced as to result in slicing of individual minerals it passes under the category of granulation. Where carried on at depth there is always some degree of cementation by recrystallization. Deformation by such close-grained fracture without complete loss of cohesion is classed as rock flowage. It is thought to be developed to some degree within the lithosphere, especially by great horizontally compressive forces, but is not

regarded here as the mode of deep rock flowage involved in the isostatic readjustment of unfolded tracts.

Thirdly, flowage may take place in some minerals, as calcite and ice, by gliding upon the cleavage planes. But such gliding is not regarded as the mode by which the foliated rocks are developed. It requires furthermore far greater force than that which is given by the departures from isostatic equilibrium.

A fourth mode of rock flowage is by recrystallization. It is the chief factor, as Van Hise has shown, in the deformation of the crystalline foliates. It is thought that this is also the method by which the asthenosphere yields and that a readiness of recrystallization under unbalanced stresses of a permanent nature is the cause of the weakness of the asthenosphere.

The vibratory forces transmitted as earthquake shocks and those due to tidal strain, from this standpoint, are both rapid. Under such conditions the asthenosphere could show high order of strength. It is argued that the ease of recrystallization under constant strain becomes more marked the nearer the temperature approaches to that of fusion, or to express it better from the physico-chemical standpoint, the nearer the temperature approaches to the mutual solution point of the constitutents involved. The result is that at such temperatures the rigidity may be high and not greatly different from that at low temperatures, but for permanent stresses the elastic limit becomes low. The movement of continental glaciers with a low surface gradient, accomplished by recrystallization, illustrates the condition which it would appear exists to even a higher degree within the asthenosphere.

This conclusion carries with it the idea that within the lithosphere the temperature is in general considerably below that of fusion; whereas below, in the thick zone of weakness, the temperature must lie close to, or at, that of molten rocks. For fusion there is needed, however, the energy necessary to supply the latent heat and volume expansion. Unless this energy is supplied, the asthenosphere remains solid rock, but the least accession of internal heat, or relief from external pressure, will generate a proportionate amount of magma, diffused as liquid throughout the solid. To gather into reservoirs temporarily molten, the magma must con-

verge by rising, analogous to the draining of melted water from glaciers; uniting, as rivulets unite into rivers, and rivers discharge into lakes. No continuous lava stratum or large reservoirs of lava could, under the terms of this hypothesis, be expected to exist within the asthenosphere. Its very weakness would prevent it from acting as a containing vessel for holding large volumes of any fluid which, for any cause such as a lower specific gravity of the fluid phase, would tend to rise. The evidence of earthquake vibrations and of resistance to tidal deformation further supports the view that the asthenosphere is not a liquid or even a truly viscous zone. On the other hand, only in the lithosphere would be found the strength needed for the storage of magma in volumes until the limit of its strength as a containing vessel was reached.

Partly guided by observation upon the metamorphic rocks, partly by theories of the nature of deformation at great depths, the argument leads to conclusions on the mode of yielding within the different levels of the crust. First, the outermost zone is observed to be a zone of fracture, weak in comparison with the thick zone below. This, the second zone, is the zone of strength and yields by flowage, but flowage which is characterized by granulation as the dominant, by recrystallization as the subordinate, mode. The expenditure of energy for a given deformation is here a maximum. In the third zone, the asthenosphere, on the contrary, flowage is conceived as taking place with but little expenditure of energy, by a ready recrystallization at the temperature of primary crystallization of magmas. Those contorted granite-gneisses seen especially in the Archean rocks, which are regarded as deformed during the final stages of crystallization, exhibit locally in the outer crust the conditions which it appears may permanently prevail within the asthenosphere.

SECTION A

RELATIONS BETWEEN RIGIDITY, STRENGTH AND IGNEOUS ACTIVITY

DISTINCTIONS IN PHYSICAL PROPERTIES RELATED TO STRENGTH

Elasticity is of two natures: that of volume and that of form. The first is possessed by matter in either the gaseous, liquid, or solid state; the second is possessed by solids only and is associated

with rigidity and strength. Up to a certain degree of strain known as the elastic limit, elasticity of form in the ideal solid is perfect and is expressed by the law that the change of form, or strain, is directly proportional to the load applied, or stress. This load may be maintained indefinitely and, except for a slight relaxation, the solid shows no further yielding. Upon the removal of the load there is an elastic return to the original form, but the very last stages of the recovery are slow. The elastic nature of the whole earth, in regard to both volume and form, is shown by its capacity to transmit the several kinds of earthquake vibrations. The permanence and perfection of the elasticity of form is also implied in the power of the crust to carry loads up to certain limits for times reaching into geologic periods without exhibiting progressive viscous yielding.

Beyond the elastic limit, elasticity ceases to be perfect, and a permanent change of form occurs. This relieves part of the stress and reduces the strain to within the elastic limit. The change of form may be by rupture, in which event the strength of the body is destroyed. It may be by plastic flow, in which case the strength may be increased or decreased. Wrought iron, for example, becomes somewhat stronger as a result of forging. Granite on being mashed into gneiss becomes somewhat weaker because of the development of weaker minerals, especially the micas. In the crust of the earth, except for the outer few miles, flowage takes place without probably much change in the mineral composition and consequently in the strength of the rock. Deformation will continue as long as the stress is maintained well above the elastic limit, but upon the cessation of movement there may still remain residual stresses up to the elastic limit. If the residual stresses over broad areas are small, it may be because the development of weaker structures, such as folds or zones of igneous injection, has eased the strains.

Failure by flow brings in the distinction between viscosity and plasticity. These are often used, even by physical geologists, as merely synonymous terms, but there is a real distinction which should be noted. Fluids are viscous to a small or large degree and can have no elasticity of form. Viscous flow must, however, overcome internal friction and requires time for its accomplishment.

With long time even a minute force will cause even a very viscous fluid to flow. Solids, on the other hand, possess elasticity of form, and below the elastic limit can hold shearing stresses indefinitely. Above it they may flow and in so doing exhibit plasticity. The phenomenon differs from viscosity in that the force must rise to a certain magnitude before any gliding between molecules begins. The crust, then, is plastic but not viscous.

Although the theoretical distinction between plasticity and viscosity is clear, recognition must be given to conditions where the two states merge. This is especially true for undercooled glasses. A glass in its molecular organization is a liquid and yet it possesses definite elastic moduli and elastic limit. From this standpoint of elasticity the glass, therefore, is a solid. Upon rise of temperature there is, however, no absorption of latent heat to mark a change of state, the elastic limit gradually lowers, disappears for prolonged stresses, and elastically, the substance passes by gradation from a solid to a liquid. The existence of these transition cases should not, however, be permitted to obscure the real distinctions between solid and liquid.

The crust yields as a plastic solid to forces which strain it beyond its elastic limit. But the solid flowage which this implies may be either by distortion of crystals or by recrystallization. The first is familiar for rapidly applied forces, requires comparatively great stress, and corresponds to the usual conception of plasticity. The crystalline rocks make us familiar, however, with the idea of mass plasticity by recrystallization. This is plasticity, but in a somewhat different sense from that which is usually conveyed by the term.

The degree of elasticity which a substance may exhibit is a different property from the elastic limit. A bar of wrought iron one square inch in cross-section will be elongated one part in 28,000,000 by a tensile stress of one pound. A similar bar of glass would be elongated one part in 10,500,000, more than twice as much. These ratios measure the degree of elasticity under tensile or compressive stresses and differ for each substance. The figure is known as Young's modulus of elasticity. A substance may be highly elastic, that is, have a high modulus of elasticity, as cast iron, or glass,

and yet be brittle because of a low elastic limit under rapid tensile stress, combined with lack of plasticity at ordinary temperatures. At temperatures sufficiently high, the modulus is not greatly different, but the elastic limit is still lower. The substance is now, however, plastic, rather than brittle, since plasticity is greatly increased; but a rapid strain, exceeding the rapidity with which plastic deformation can take place, may still produce fracture. Another substance, such as rubber, may have a low modulus of elasticity and yet a relatively high elastic limit.

Among similar substances under similar physical conditions there is, however, a definite association of these properties which for the metals is brought out well in a tabulation by Johnston and L. H. Adams.[1] It is shown for a class of substances, such as the metals, that the modulus of elasticity, the hardness, the tensile strength, and the elastic limit all, so far as the data are given, occur in the same order; so that of two metals that which has the higher elastic limit is the higher also in the other qualities. From this association there results a ready mental confusion between rigidity and strength. The one, however, denotes the degree of resistance to distortion from a unit-shearing stress and gives the modulus of ridigity. The other is measured by the elastic limit. As an example of the confusion between these two different properties, it is known that the earth as a whole is more rigid than steel. This to many would appear to mean that it was stronger than steel. Earthquake waves show that the earth becomes progressively more incompressible and more rigid with depth. This might be held as evidence against the existence of a thick sphere of weakness, the asthenosphere. High incompressibility and high rigidity are not, however, direct testimony of strength, and it is the purpose of the next topic to show under what conditions a solid may be very rigid and yet very weak.

CONDITIONS FAVORING ASSOCIATION OF HIGH RIGIDITY WITH LOW ELASTIC LIMIT

Alpine glaciers as well as the Alpine-like margins of the Greenland ice sheet move much more rapidly in the summer than in the

[1] "On the Effect of High Pressures on the Physical and Chemical Behavior of Solids," *Amer. Jour. Sci.*, XXXV (1913), 220.

winter, a phenomenon to be accounted for by the rate of recrystallization. The parts of an ice crystal which are subjected to shear and compression have the melting-point lowered. They melt, discharge the strain, and refreeze. In the winter the general temperature is reduced, and a greater strain is necessary to bring the melting-point down to the lower temperature. Until local melting is produced the ice behaves like any other crystalline solid, as a substance possessing elasticity of form. Beyond that point it exhibits plasticity and behaves in some respects like a very viscous fluid. In other respects, however, it exhibits properties quite distinct from that of the usual conception of mere plastic flow, since in the testing machine, or on the walls of a crevasse, ice will resist strong shearing strains, and yet the glacier as a whole yields an flows slowly under a moderate pressure-difference as shown by the low gradient of its upper surface. Glacial motion appears to take place, therefore, by the solution and growth of crystals, not by a true viscous flow. The solid and crystalline nature throughout as opposed to viscous fluidity is furthermore shown, as Chamberlin has noted, in the power of the glacial ice to shove over and abrade its floor and to ride up slopes. Chamberlin adds that a dry glacier is a rigid glacier. A dry glacier is necessarily cold, and a cold glacier is necessarily dry.[1]

With ice subjected to slowly applied forces the elastic limit is consequently dependent upon the point of yielding by recrystallization. We thus see an intimate relationship between temperature and variation in the elastic limit, the elastic limit for ice being greater for low temperatures than for high temperatures. But the modulus of rigidity, on the contrary, measures the elastic change of form for unit-shearing force, change of form not accompanied by crystallization, but marked by a capacity to spring back to the original form upon the removal of the stress.

T. W. Richards, in his studies on the compressibility of solids, notes that they are almost as compressible and voluminous at absolute zero as at ordinary temperatures. Under this conception

[1] "A Contribution to the Theory of Glacial Motion," *Decennial Publications of the University of Chicago*, IX, 203, 204 (1904); Chamberlin and Salisbury, *Geology*, I (1904), 305.

the molecules, taken as equivalent to their spheres of influence, are in actual contact and suffer mutual compression owing to the attraction of cohesion. The influence of heat is relatively unimportant in determining the density of a solid. The atoms are in most cases even more compressed and distorted by the converging force of chemical affinity than are the molecules by cohesion. This corresponds with the fact that substances of small atomic volume are on the whole more incompressible than those of greater atomic volume. For the more incompressible substances also the decrease in compressibility with added load is relatively little, suggesting that they are already greatly compressed by the forces of chemical affinity.[1] The effect of heat serves only to distend slightly the spheres of influence of the atoms, so long as the substance is in the solid state. Rise of temperature to near the melting-point, as long as there is not a softening by the development of incipient liquidity, should, according to these views, change the elastic properties but slightly. The greatly lessened strength of ice near the melting-point, as expressed in the freedom of regelation, is not, following these ideas, connected with the slightly lessened incompressibility and rigidity. For many substances the problem is complicated, however, by changes of molecular state with changes in temperature and pressure. This is especially true of ice when subjected to extreme ranges in temperature and pressure, as has been shown by Bridgeman; for ordinary glacial ice, however, we deal with but a single state.

For ice at a temperature of $-7^{\circ}.03$ C. the compressibility has been determined for pressures ranging approximately between 100 and 500 atmospheres. It is found to possess, according to Richards and Speyers, about one-fourth of the compressibility of water at neighboring temperatures and about five times the compressibility of glass.[2] But glass possesses a compressibility between that of acidic and basic holocrystalline igneous rocks. Ice may be taken then as about three or four times as compressible as granite.

[1] "The Present Aspect of the Hypothesis of Compressible Atoms," *Am. Chem. Soc. Jour.*, XXXVI (1914), 2417–39.

[2] T. W. Richards and C. L. Speyers, "The Compressibility of Ice," *Am. Chem. Soc. Jour.*, XXXVI (1914), 491–94.

Now the modulus of rigidity is related to the modulus of compressibility by means of a formula involving Poisson's ratio.[1] This ratio varies for each substance, but for rocks, for iron and steel, and probably for ice, it lies between 0.2 and 0.3 in value, so that in general the rigidity of these substances can be judged roughly by their degree of incompressibility. Consequently it is seen that glacier ice at temperatures such as those which prevail in the body of the moving glacier possesses a degree of incompressibility and rigidity which, if these elastic constants were measures of its strength, would make it wholly incapable of motion on such gradients as are observed. This can be made more obvious by some quantitative statements. Granite and similar rocks, for example, can stand permanently in steep cliffs to heights of thousands of feet. They constitute mountain ranges whose height and steepness are limited entirely by the forces of erosion on the one hand and the strength of the asthenosphere on the other. The cliffs could be very much higher and the mountains much more lofty before glacier-like flow at the base of the mountain mass would occur. In fact, with a compressive strength of 25,000 pounds per square inch, a rectangular block of granite could stand as a vertical wall 22,000 feet high, and of indefinite breadth, without yielding of the base. With a sloping face and supported by spurs such as occur in nature, the height of the granite mass could become considerably greater. For parallel mountain ranges of harmonic form and gentle slopes resting upon a foundation whose compressive strength to indefinite depths was 25,000 pounds per square inch, the mountain crests could stand eleven miles above the valley bottoms before the maximum stress-difference would reach this limit. Even then, if the slopes were as low as those of a continental ice sheet, the failure would not take place by flowage of the mountains laterally into the valleys, but by a vertical settling of the mountains and a vertical upwarping of the valleys. The lateral, plastic flow would be at some depth in the earth. If the asthenosphere were indefinitely rigid, granite mountains of sufficiently gentle slope

[1] Let P=Poisson's ratio; C, the modulus of rigidity; D, the modulus of compressibility. Then $C=\frac{3}{2}\,\frac{1-2P}{1+P}\,D$.

could rise to indefinite heights. This is because the depth of maximum stress-difference would lie at about one-sixth of the wave-length below the mean level of the surface. With increasing wave-length the height of the waves could accordingly be greater without increasing the stress-difference at the trough-line of the waves. The gradient would, however, have to become more gentle; in other words, the amplitude would have to increase at a lesser rate than the wave-length. If the strength of ice were measured by its rigidity it could stand permanently in masses one-third or one-fourth as steep and high as these theoretic limits for granite mountains, without failure by plastic flow. Yet, on the contrary, the great ice fields spread out by flowage of their bases, although their surfaces possess very gentle gradients. The distinction between strength and rigidity in the movement of glaciers is thus clear. The strength of glaciers is limited by the amount of the stress-differences needed to produce slow movement by recrystallization.

Johnston and L. H. Adams have applied this theory of yielding, well known as an explanation of glacial motion, to all plastic flow, and argue that even for those substances, such as the metals and rocks, in which cubic compressibility raises the melting-point, shear greatly lowers it for the parts under stress.[1] They argue from a physico-chemical basis that the most plausible explanation for flow in metals is that the shearing strain is great enough on individual points to produce a change of phase of individual molecules from solid to liquid, even at ordinary temperatures.

Apart from theory as to its explanation, the phenomenon of welding of iron shows for high temperatures a low elastic limit and ready passage beyond into plastic flow. For iron and steel, furthermore, the influence of temperature upon the rigidity has been investigated. Pisati gives the following equations in which n is the value of the modulus of rigidity for temperature t.[2] For iron—

$$n_t = 811 \times 10^6 (1 - .000{,}206t - .000{,}000{,}19t^2 + .000{,}000{,}001{,}1t^3),$$

[1] "On the Effect of High Pressures on the Physical and Chemical Behavior of Solids," *Am. Jour. Sci.*, XXXV (1914), 205–53.

[2] *Smithsonian Physical Tables* (1904), p. 76.

for steel—

$$n_t = 829 \times 10^6(1 - .000{,}187t - .000{,}000{,}59t^2 + .000{,}000{,}000{,}9t^3).$$

This equation for iron gives a minimum modulus at 314° C. equal to 95 per cent of the modulus at 0° C. For steel the minimum value occurs at 342° C. and is 90 per cent of the modulus at zero. At 528 °C. iron has the same modulus as at 0° C. and at 890° C. steel has the same modulus as at zero. Doubtless 1000° C. is above the limits of the data from which these formulas were derived. For this temperature they may consequently give inaccurate results, but it is of interest to note that the curve gives a modulus of rigidity for iron at that temperature 1.7 times that at 0° C. and for steel 1.1 times that at 0° C. The extrapolation prevents attaching quantitative value to these figures, but the qualitative conclusion may be reached that iron and steel at high temperatures do not exhibit less rigidity than they possess at lower temperatures. It is obvious, however, that above a certain temperature the elastic limit becomes very low, as shown by the capacity for forging, and for strains beyond this limit deformation takes place by plastic flow. That it is not merely incipient fusion is suggested by the maintenance of a crystalline condition through the process of deformation. The subject for iron is doubtless complicated by the fact that iron passes through more than one solid molecular state in being heated up to fusion. Presumably then the equation given for the relation of rigidity to temperature can only be a first approximation to the actual changes.

Let the attention be given next to the crystalline rocks which were once deep-seated and, owing to subjacent batholithic invasion, attained their crystallization at exalted temperatures. It is observed that, although the rock masses have been extensively deformed, the individual crystals have regrown during the process so as to possess compact boundaries, and an internal constitution nearly free from strain. The interpretation is that the deformations due to geologic forces were so slow and the rocks were so saturated with crystallizing agents at high temperatures that recrystallization could nearly keep pace with the deformation, even for temperatures below the range of plasticity. As understood by

the students of anamorphism, the process has depended upon a readier solution of the molecules under shearing stress than of those free from such stress—solution carried on by means of the relatively minute proportion of gaseous crystallizers which were present through the rock mass. Such crystallizers doubtless facilitate the process. They form, in fact, solutions with the rock which may be regarded as mixtures with very low fusion points. But, theoretically, as the temperatures approach those of general fusion the need of such crystallizers diminishes. Moderate shearing stresses can thus liquefy the parts upon which they act and a process analogous to glacial motion sets in for solid rock. The strength of highly heated rock appears then dependent upon the amount by which the temperature is below the melting-point. That zone of the earth which is very weak may then be regarded as approximately at the temperature of fusion. To transform the solid into liquid there is needed only the energy required for latent heat and increase of volume. The proportion of liquid which is generated will vary directly with the amount of heat supplied or the amount of hydrostatic pressure removed. Magma, consequently, can be generated in this zone more readily than above in the zone of strength; but it will not be in reservoirs; rather will it be in its place of origin disseminated through the rock mass like water standing in a porous sandstone.[1]

ANALOGIES BETWEEN ASTHENOSPHERIC ROCK AND GLACIAL ICE

The theory of the asthenosphere as here presented is seen to have important relations to other branches of geology. The zone of weakness becomes especially the generator of magmas; the

[1] Recently the writer has learned from Mr. Bailey Willis of an unpublished paper which he gave some years ago to the Geological Society of Washington, in which he outlined his views of the nature of crustal thrusts as illustrated by the Appalachians. In that, and more recently, as a result of studies in the Alps and Andes, he has come to entertain the view that the zone of compensation, the lithosphere, shears over the zone below through the agency of molecular or mass fusion. Deep-seated horizontal shear and igneous intrusion he thus holds have important associations with orogenic movements. We have thus arrived independently at somewhat the same view of the nature of the zone of weakness. The part which recrystallization may play in promoting such movement is suggested in his *Research in China*, Vol. II (1907), "Systematic Geology," pp. 130, 131.

pyrosphere has its roots in the asthenosphere. But in the attempt to frame a logical picture of the processes which determine the ascent of magmas, the question arises how diffused liquid matter is drained away, rising and uniting at higher levels into magma reservoirs, temporarily molten, and the direct source of the igneous activity exhibited within the outermost crust. A deductive picture is as follows—one whose truth cannot be tested directly, but only by its general agreement with our understanding.

At the place of origin, liquid of an andesitic or basaltic nature will come to honeycomb the rock. The content of gases is presumably sufficiently high to reduce the viscosity. The liquid will then become able to transmit hydrostatic pressures, and, although comprising only a part of the rock mass, will constitute a continuous column of considerable height. Then becomes possible the second stage, the draining upward and the convergence of the fluid rock. Gravity is the ultimate cause, as in the downward movement of waters, but here the fluid, being lighter than its surroundings, tends to move upward. An explanation of this draining process has been given by Lane.[1] In a gas-saturated rock an excess of gas, or liquid and gas, would have the power of opening fissures at any depth in the zone of flow without the necessity for the existence of any tensile stress in the walls, or competence in the walls to maintain an open cavity. All that is necessary is that the excess pressure in the rising wedge of gas should be stronger than the cohesion of the rock. The fissure becomes filled with gas and fluid of lesser density than the solid rock of the walls. Consequently, the pressure transmitted from below is greater than the resisting pressure in the walls. This insinuating power, owing to the hydrostatic head due to the lesser gravity of the wedge, becomes greater the higher the wedge rises above the source, until near the surface the action may become violent and rapid. Daly also has outlined a theory of mechanism for the injection of abyssal wedges of magma into the upper crust.[2]

[1] "Geologic Activity of the Earth's Originally Absorbed Gases," *Geological Society of America Bull.*, V (1894), 259–80.

[2] *Am. Jour. Sci.*, XXII (1906), 195–216; *Igneous Rocks and Their Origin* (1914), chap. ix.

His theory is constructed however for action within the roof of a magmatic substratum. Daly postulates a zone of tension, but the mechanism suggested by Lane does not require this and would seem to apply better to the region of generation of magmas, for there the cubic compression is enormous, it is not a zone which has been subjected to cooling, and therefore it is difficult to conceive of the cause of a system of tensile stresses within the asthenosphere.

We are now prepared to draw a closer analogy between the physical conditions of the asthenosphere and those of a glacier, noting the likenesses and also the unlikenesses. In the summer, in the case of Alpine glaciers, heat is supplied to the surface of the glacier until it is warmed to the melting-point, and part of the ice absorbs the amount required by the change of state and passes into water. This trickles along the surface until a fissure is met and the water sinks by force of gravity toward the bottom. Near the snout of the glacier the temperature of this deeper part may thus be more dependent upon this convection than upon direct conduction through the ice. The winter freezing tends to chill the deeper ice and slow its motion, but during the summer the descending water tends to raise the temperature toward the freezing-point. In parts of the glacier where the heating is more effective than the cooling, the waters drill channels and gather at the base of the glacier into streams, reaching finally the outer world. The descent, and gathering, and englacial flow of glacial waters is analogous to the rise and convergence of streamlets of molten rock.

In order to account for lateral mass movement within the asthenosphere, an imperfection of isostasy between continental interior and ocean basin, giving an isostatic gradient or slope toward the continental interior, seems a necessary postulate. Such a gradient is so low that it has not yet been sifted from those irregularities of mass which are owing to the strength of the outer crust, or, it may be, obscured by great compressive bowings of the crust. This isostatic gradient, the slope required to generate movement within the asthenosphere, is far lower than that of the surface of a continental ice sheet. The failures of the analogy between the asthenospheric and glacial states are then as instructive as the agreements. The glacier is thin and broad. Friction on the bot-

tom is excessive and the motion requires more internal work. Much of its mass is permanently well below the freezing temperature. These are the factors which determine the steepness of the surface gradient. The asthenosphere by contrast should be deep and the differential motions within it necessary to satisfy isostasy would be correspondingly small. The temperature through a wide zone should be that of fusion under the hydrostatic pressures prevailing. The solid rock should be sodden with occluded gases, giving mobility to the growing fluid and ready to play their part in assisting recrystallization. Such a physical condition, as long as there is a continuous solid, would exhibit perfect elasticity and high rigidity during the passage of transverse vibrations, yet would slowly yield to prolonged shearing stresses, even though these were very small in amount.

RELATIONS OF IGNEOUS ACTIVITY TO ASTHENOSPHERE AND LITHOSPHERE

The argument has led to the view that the asthenosphere is a region where the temperature curve becomes tangent to the fusion curve, but that a condition of solidity is maintained by the recurrent elimination of that material which becomes molten. The importance of such a process, maintaining the solidity of the earth, has been dwelt upon by Chamberlin, especially as accounting for the overwhelming igneous activity of Archeozoic time. In lessened measure it applies to all later times as well.

Becker has held that the bottom of the zone of isostatic compensation is the depth at which the temperature curve approached nearest to the fusion curve, and he was the first to connect in this way the geodetic evidence with a temperature relation.[1] But Becker does not conceive of actual, permanent contact of the two curves as occurring, and took the depth of nearest approach to fusion as 122 km. This follows from Hayford's hypothesis of a uniform distribution for isostatic compensation, but in the present work there has been found reason for believing that compensation fades out through a greater depth; the strength, as measured by

[1] "Age of a Cooling Globe in Which the Initial Temperature Increases Directly as the Distance from the Surface," *Science*, XXVII (1908), 227–33, 392; "The Age of the Earth," *Smithsonian Miscellaneous Collections*, LVI, No. 6 (1910), 1–28.

the existence of stress-difference, through a depth greater still. The beginning of permanent contact of the two curves, if this is the cause of the disappearance of strength, should be as much as 300 km. deep and extend through some hundreds of kilometers.

A rectilinear projection downward of the temperature gradient observed at the surface would reach the fusion temperature of rocks at a depth of about 50 km. There must be consequently a marked curvature of the temperature gradient if the temperature and fusion curves do not meet short of 300 km. This curvature implies that near the surface there is either a greater quantity of heat flowing outward by conduction or that the conductivity of rock near the surface is very greatly decreased. But such a very great decrease in conductivity making for a higher temperature gradient finds no supporting evidence. On the other hand, a greater outward flow by conduction of heat near the surface may be due to the continued generation of heat by radioactivity to a greater degree than below; or also to a rise of magmas from the asthenosphere. Magmas which never reach the surface would bring heat by a convective process directly into the outer crust. From there the heat, slowly diffused upward by conduction, would increase the temperature gradient in the outermost part of the lithosphere. It is this factor especially which the argument of the present chapter emphasizes.

It is only within the present generation that general recognition has been given to the intrusive nature of the abyssal igneous rocks. They are now generally regarded as risen from the depths. Their action has been to break through and engulf the foundations of the ancient crust.

This process of batholithic invasion seems to be recurrent and widespread, though rise into the outer crust is restricted to the crises of diastrophism and usually reaches levels exposed to erosion only along the lines of mountain systems. The stores of heat brought up from the greater depths would be held in the crust, especially in its deeper parts, for geologic ages, blurring out in the course of time by conduction and creating a false appearance of heat lingering from an initial molten state, a resemblance increased by the added veil of new heat of radio-active origin mantling the ancient stores.

The temperature gradient under this view should naturally vary widely from place to place and from time to time. Igneous activity is the effective means by which heat is brought up from depths which on account of the slowness of conduction would be otherwise thermally isolated from the outer crust. Offset against this, cooling by conduction advances downward from the surface, dissipating not only the heat of local radio-active origin but that excess rising from the depths. The heat of the crust is not then a continually ebbing residuum from a primal molten state, but represents rather an oscillating ebb and flow, one of the balances of nature maintained through geologic time.

If this view be true—that the invasive igneous rocks have been an important factor in determining the amount and distribution of heat in the crust—it is doubtful if any sound arguments can be derived from the study of the present gradients as to the initial temperature. This conclusion is similar to the change of viewpoint in other lines of geology. It was once thought that the composition of the present atmosphere and the character of present climates were steps in a simple and continuous series of changes passing from primal conditions to a future in which the water would be absorbed into the earth and its surface transformed into a frozen desert. Now, however, it is generally recognized that since the earliest known times the surface conditions have been in a state of oscillating equilibrium. The argument of this section leads toward the view that this is true for the physical conditions within lithosphere and asthenosphere also.

[*To be concluded*]

THE STRENGTH OF THE EARTH'S CRUST—*Concluded*

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PART VIII. PHYSICAL CONDITIONS CONTROLLING THE NATURE OF LITHOSPHERE AND ASTHENOSPHERE

SECTION B

RELATIONS WITH OTHER FIELDS OF GEOPHYSICS

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RELATIONS WITH OTHER FIELDS OF GEOPHYSICS

Erroneous Conclusions Reached by the Rectilinear Projection of Surface Conditions

Geologists early became aware that temperature increased with depth. Projecting this gradient as a straight line indicated that at no great depth the temperature was sufficiently high to melt all rocks and, in testimony, volcanoes brought such melted rocks to the surface. The earth was consequently looked upon as a molten or even gaseous body enveloped by a thin crust of solid rock. The logic of this conclusion seemed incontrovertible and moreover it was in accord with the simpler expectations from the nebular hypothesis. Nevertheless, direct and positive evidence from several independent sources has forced on geologists the belief that the earth is not only solid throughout, but, as a whole, is more rigid than steel. Slowly and with difficulty the older view has therefore had to be abandoned. Yet it continually recurs in one form or another, advocated chiefly by writers who see the direction in which the surface evidence of temperature gradient leads, who regard it as compulsory, and who do not recognize or give equal

weight to the direct evidence regarding the nature of the earth's interior. Because of the ease and certainty of laboratory studies there is a tendency to treat the interior of the earth as though it were incapable of speaking for itself through the evidence of geophysics, geodesy, and geology, but must remain forever a playground for the speculative imagination. Largely unknown the nature of the earth's interior is and long must be; laboratory studies on the influences of heat, of pressure, and of chemical composition, upon the physical state of the crust, must constitute the paths which guide in the search downward into the unknown; but the final test of hypothesis must be the direct testimony of the earth itself.

The rectilinear projection of surface conditions is based on the assumption that the temperature gradient is a straight line to great depths, or that strength, or density, or porosity, as the case may be, is not changed by the pressures of the interior. Such assumptions lead to views more or less in opposition to those reached in the present investigation. They must, therefore, be discussed to some degree. An illustration of these dangers of reasoning by unchecked extrapolation is supplied by a paper written by Arrhenius,[1] selected for discussion because of the eminence of the author in the fields of physics and chemistry, the definiteness with which his conclusions are stated, and the wide citation which this paper has achieved.[2] In this paper the arguments are given in favor of a gaseous nature of the interior of the earth, carrying forward an idea first suggested by A. Ritter in a series of "Researches on the Height of the Atmosphere and the Constitution of Gaseous Heavenly Bodies."

From the rate of increase of temperature with depth Arrhenius argues that at a depth of 40 km. the crust must pass into a molten condition, but one which, because of pressure, is a viscous and highly incompressible liquid. At a depth of some 300 km. the temperature, he states, must be above the critical temperature of

[1] "Zur Physik des Vulkanismus," *Geol. Foren. i Stockholm, Forhandl.*, XXII (1900), 395–419.

[2] See for example its presentation by A. Geikie, *Text Book of Geology*, Vol. I (1903), pp. 71–74.

all known substances, and therefore the liquid magma passes into a gaseous magma extending to the center of the earth. The author then notes that the chemical elements of highest atomic weight are not detected in the sun, but states that without doubt they occur, and concludes from this that they must be concentrated by virtue of gravity toward the sun's center.[1] The high density of the earth's interior is accordingly to be explained by the presence of substances heavier than surface rocks. For many reasons, as the dominance of iron in nature, as shown by meteorites, by the spectrum of the sun, and by the magnetism of the earth, it is to be concluded that this substance which he thinks necessary to account for the high density of the earth's interior is metallic iron. The earth consists consequently of the following portions measured from the center on the radius. Eighty per cent of the radius is gaseous iron, 15 per cent is gaseous rock magma, about 4 per cent is fluid rock magma, and somewhat less than one per cent is solid crust.[2]

To reconcile these conclusions with the incontrovertible evidence of rigidity, Arrhenius takes up another line of rectilinear extrapolation and carries it to an equally extreme degree. Fluids in general show a somewhat readier compressibility than solids. At high pressures then it is argued that liquids will customarily occupy less volume than solids and the pressure will tend to lower, not raise, the melting-point. Consequently, the rigidity cannot be accounted for by the maintenance of solidity through pressure. The author then points out that under enormous pressures all substances, even gases, must become highly incompressible; and that at high temperatures, where the volume is maintained the same, the viscosity of gases or fluids increases with increase in temperature. From this it is argued that in the central parts of the earth gaseous iron is more incompressible and viscous than solid steel. It is by enormous pressure consequently in spite of a gaseous nature that the interior of the earth exhibits its great rigidity.

Vulcanism according to Arrhenius is connected with the free seepage of ocean water downward through the crust which, he holds, constitutes a semipermeable membrane. By the absorp-

[1] *Op. cit.*, p. 402. [2] *Op. cit.*, p. 405.

tion of water into the heated rocks the conditions for volcanic activity are initiated. This argument, like the others, is in the form of a great extension or extrapolation of factors operative in a small way in the laboratory to conditions in nature which are wholly different in magnitude.

As comments upon this paper, it should be noted that nearly every conclusion applying to the sun and earth may be questioned.

At a depth of 1,000 km., according to Arrhenius, the temperature is about 30,000° C. The gradient is thus taken as essentially a straight line from the surface downward. There is no demonstration as to why this rectilinear extension is assumed, whether it is to be regarded as an adiabatic temperature curve produced by condensation under pressure or produced in some other way. The influence of cooling through geologic time in changing the outer gradient is not considered; nor the influence of rising magmas. The existence of radioactivity was then just beginning to be appreciated and naturally could not have been evaluated, but the data for a discussion of the other factors, though at hand, was neglected.

There is no demonstration that the heavy elements are concentrated in the sun's interior, or that the earth is mostly metallic iron. It is possible that the earth is thus constituted, but it must be proved on better evidence than a citation of the dominance of iron in nature. The incompressibility of all substances, both fluid and liquid, increases greatly with great increase of pressure, following apparently parabolic curves. Therefore, it cannot be argued with any assurance that the high incompressibility of the earth's interior proves the presence of iron, or that under such pressures the fluid occupies less volume than the solid state.

At a depth of 1,000 km. Arrhenius states that the temperature is about 30,000° C. and the pressure 250,000 atmospheres.[1] If, under these conditions of exalted temperatures, gaseous rock or iron has a viscosity equal to that of solid steel it may well be asked how the stars, with their immensely greater masses and consequent internal pressures, can maintain a convective circulation competent to keep up their enormous surface radiation. Furthermore, however viscous a compressed gas or liquid may be, this property

[1] *Op. cit.*, p. 400.

should be distinguished from rigidity. If a body can resist even small shearing stresses for an indefinite period, it has the essential properties of a solid and not a gas. If it possesses real rigidity, even if it should be true that under relief from pressure the substance would turn into a gas, yet such relief cannot take place and it is a confusion of terms to speak of the substance as a gas when exhibiting to a striking degree the essential qualities of a solid. This distinction between viscosity and rigidity is of first importance, yet is not mentioned by Arrhenius. Although undercooling of a fluid into a glass gives rise to the elastic properties of a solid, it has not been shown that increase of pressure, however great, upon a gas above the critical temperature would transform increasing fluid viscosity into solid rigidity and plasticity such as is exhibited by the earth.

As to the hypothesis that the crust is a semipermeable membrane, permitting a free downward seepage of ocean water, but little need be said, since this is a subject which has been much discussed in recent years and is now largely discarded by geologists. The evidence against it is varied. Petrologic study shows the deep rocks to be impermeable and unaltered; beyond a shallow depth they are dry, and their gaseous and liquid occlusions are held unchanged for geologic ages. Unsound conclusions have been built upon the behavior of steam within porous sandstones, combined with confusion of the rate of diffusion under enormous pressure-gradients in the laboratory with enormous pressures, but low pressure-gradients within the crust. Furthermore volcanoes are not restricted to the vicinity of the sea and their emanations are not of the proper composition to have been derived from ocean waters. As Suess has said, volcanoes are not nourished by the sea, but every volcanic eruption adds to the waters of the ocean.

The paper under discussion was written by a scientist who has done much exact work in physical chemistry, but who in passing to geologic thinking has adopted the habit of an earlier generation —a habit of speculative thought, suggested by chemical and physical concepts and not verified by a study of the earth. The form of present geologic investigations has, however, advanced to the quantitative stage, although the data are often so inexact that the

order of magnitude, or the direction of the truth, is all which may be now ascertainable.

In conclusion, it is seen that the hypotheses outlined by Arrhenius imply a thinness of the crystalline lithosphere and a crustal weakness wholly at variance with the conclusions regarding strength which have been reached in this investigation. They imply a difference in nature of the earth's interior from that given by the more direct lines of evidence, as shown by the body resistance of the earth to vibratory distortions of both short and long periods. Because of these many difficulties, this group of hypotheses, adopted by Arrhenius, has already been largely discarded, though they still find considerable acceptance, more especially by workers in related fields of science. But the measures of lithospheric depth and strength which appear to be given by geodesy add their testimony to the cumulative evidence against these views.

THE EVIDENCE OF TIDES ON RIGIDITY AND STRENGTH

The tidal distortion of the solid earth measured by means of the horizontal pendulum has shown that its rigidity is of the order of magnitude of steel. But the recent measurements by Michelson and others, employing a long horizontal pipe partly filled with water, showed clearly that the earth's rigidity is even greater than that of steel.[1] This higher value is in agreement with the inductions from the observations on the variations of latitude. But these measurements give the rigidity of the earth as a whole, not the distribution of rigidity. The resistance to tidal deformation is furthermore complicated by the influence of gravity and increasing density with greater depth. Even if the earth were a liquid globe it would resist tidal distortion to one-third the degree of the resistance of a globe of steel, and if the liquid sphere were denser inside, this ratio would be further decreased.[2] Notwithstanding this factor, however, it is clear that the earth as a whole is more rigid than steel. As the outer part is known to be less rigid than steel, it follows that the rigidity of much of the interior must be

[1] "Preliminary Results of Measurements of the Rigidity of the Earth," *Jour. Geol.*, XXII (1914), 118.

[2] A. E. H. Love, *Elasticity*, p. 306.

proportionately higher. But the tidal stresses, though serving as a measure of the rigidity of the earth as a whole, are so small that they are ineffective as a measure of the strength of the earth as a whole, or of even its weakest parts. The smallness of the stresses can be appreciated by noting Darwin's numerical calculations. In his original paper Darwin arrived at the conclusion that the tidal stress-differences at the center of the earth were eight times as great as at the surface, and this result has been widely quoted. In the final publication, however, a correction is made showing that this is the ratio between the surface stress *at the poles* as compared to the center. The stresses at the poles, at the equator, and at the center he finds to be in the ratio of 1 to 3 to 8. The diurnal tide gives an actual stress-difference per square centimeter amounting to 16 grams at the poles, 48 at the equator, and 128 at the earth's center.[1] The strength of granite at the surface of the earth averages about 1,700,000–2,000,000 grams per square centimeter. The elastic limit for steel subjected to tensile or compressive stresses in one direction ranges from about 3,500,000 grams to 4,500,000 grams per square centimeter, according to the grade of the metal. The ultimate strength is about twice as high as the elastic limit. Thus the earth is stressed by the tidal forces even at the center to only about one part in fifteen thousand of the strength of good granite at the surface, or about one part in twenty-seven to thirty-five thousand of the limits of perfect elasticity which steel exhibits in the laboratory. With stresses so small it is not surprising that although tides give measurements of rigidity their evidence regarding viscosity is most uncertain. The results of estimates of the viscosity are more or less contradictory and so small as to be within the probable error of determination. Nevertheless Schweydar considers that there is a suggestion of a slightly plastic zone extending from a depth of about 120 to 620 km. Although this has been adopted in the present article as the limit of the asthenosphere, it would appear that the convincing proof for the existence of such a zone, and the determination of its limits

[1] George H. Darwin, "On the Stresses Caused in the Interior of the Earth by the Weight of Continents and Mountains," *Collected Scientific Papers* (1908), II, p. 481; original publications, *Phil. Trans. Roy. Soc.*, CLXXIII (1882), 187–223, and *Proc. Roy. Soc.*, XXXVIII (1885), 322–28.

also is more likely to be given by the geologic and geodetic evidence rather than from that yielded by the tides, provided that the present hypothesis of the existence of an asthenosphere is accepted.

It might seem that if the asthenosphere is strained to its limit by permanent stress and is slowly yielding, that even the small and rhythmic tidal stresses, like the last straw on the camel's back, might reveal a lack of resilience in the region of yielding. The distinction was emphasized in Section A, however, that an elastic limit which is determined for permanent stress by a facility of recrystallization at a high temperature may be a far lower elastic limit than that which would exist for rapid rhythmic stresses. Recrystallization would theoretically go forward a little more rapidly during the additive phase of the tidal stress, but the process is presumably so slow, and the tidal stress so small and rapid, that no appreciable effects would be attained before the following of the negative phase. A high resilience of the earth under tidal stress seems therefore quite compatible with the existence of a slowly yielding asthenosphere.

THE EVIDENCE OF EARTHQUAKE WAVES ON RIGIDITY AND DENSITY

The speed of an elastic wave through a solid varies directly with the square root of the modulus of elasticity and inversely with the square root of the density. There are two waves, corresponding to the elasticities of volume and form respectively, the one measured by the modulus of compressibility, the other by the modulus of rigidity. The first is the longitudinal or radial wave, the second is the transverse wave. The former outruns the latter and gives rise to the first preliminary tremor by which the earthquake records itself in distant regions. The transverse vibration is felt as the second preliminary tremor, followed by the much larger oscillations of the principal wave. The first two go through the earth, the latter passes around the surface. The fact that there is a transverse wave shows that the earth is solid throughout. But the vibrations at the point of emergence for waves which have penetrated more than half-way into the earth are so faint because of distance that their beginnings are in doubt, and consequently the speeds of transmission below one-half of the radius are uncer-

tain. These greater depths do not, however, so immediately concern the present subject. For the outer quarter of the earth both radial and transverse waves increase in velocity of transmission with depth, showing that incompressibility and rigidity increase faster than density and reach values greater than those exhibited by steel at the surface of the earth.[1]

So much is certain, but when it comes to testing the character of any particular shell by means of the velocities and character of the vibrations which have passed through it, there is but little certainty. The difficulty of an exact interpretation is discussed well by Knott.[2] To illustrate the variety of opinions, Benndorf has worked out a law according to which the speed of transmission increases rapidly to a depth of 200 miles (320 km.) from the surface. Knott assumes a constancy of speed below a depth of 400 miles (644 km.).[3] Wiechert has concluded that there are sudden changes in velocity at depths of 1200, 1650, and 2450 km. Poisson's ratio which expresses the relationships of the elasticities of form and volume remains, however, practically constant throughout, having a mean value of 0.27.[4] These changes imply surfaces of discontinuity. If real, however, they are deeper than the shells of the earth involved in the problems of isostasy. The conclusions rest, however, upon data of doubtful reliability. Reid has made a critical examination of this subject in connection with his comprehensive study of the excellent records obtained from many parts of the world of the California earthquake of 1906.[5] Following Wiechert's method, the curves representing the normals to the wave fronts and the velocities at various depths were computed from the data of the seismograms. The result showed that for the radial or longitudinal wave the velocity increased rapidly with depth but with decreasing rapidity, from 7.2 km. per second at the

[1] Galitzen, *Vorlesungen über Seismometrie*, p. 138, 1914.

[2] *Physics of Earthquake Phenomena* (1908), chap. xii.

[3] *Op. cit.*, pp. 248–50.

[4] G. W. Walker, *Modern Seismology*, 1913, p. 61.

[5] *California Earthquake of April 18, 1906* (Report of the State Earthquake Investigation Commission, Vol. II, "The Mechanics of the Earthquake," by H. F. Reid). Published by the Carnegie Institution of Washington, 1910.

surface to 12.5 km. per second at 2,170 km. from the surface, 0.66 of the radius from the center. Below that depth the velocity is nearly constant. The velocity of the transverse waves is 4.8 km. per second at the surface and increases almost linearly with depth, reaching a velocity of about 7.5 km. per second at half the distance to the center of the earth. The absence of good records from distances beyond 125° prevents a knowledge of the velocities at greater depths. Within the limits regarding which information is given, Reid remarks that there is no indication of a sudden change in the velocity of either wave such as we should expect if there were any sudden changes in the nature of the earth's interior. Oldham also finds no evidence of sudden change to a depth of at least 2,400 miles, 0.4 radius from the center.[1] From the curves showing the relation of velocity to depth which Reid gives[2] it is seen that the ratio of velocity of the transverse to the velocity of the longitudinal wave is 0.66 at the surface, 0.56 at 0.95 R, 0.53 at 0.9 R, reaching a minimum of 0.52 at 0.85 R, from which it increases to 0.58 at 0.5 R. This shows that both moduli of elasticity increase with depth, but that down to a depth of between 0.8 and 0.9 R. from the center of the earth, 637 and 1,274 km. from the surface, incompressibility increases relatively faster than rigidity. The change is shown as very rapid in the first 300 km. This is the only way in which the existence of an asthenosphere reflects itself in the rigidity of the earth, and this may not be related to its weakness but to some other property, such as the nature of compressibility or of changing chemical composition, or partly in the lack of detailed knowledge in the nature of the data.

Earthquake waves, like the tides, measure elasticity rather than strength. The vibrations which penetrate 200–300 km., and more, downward in the earth are already greatly reduced in amplitude and therefore in the strains which they bring on the earth. What the maximum strains may be is unknown, but reasonable assumptions as to amplitude show that within the asthenosphere the order of magnitude of the strains would be of the nature

[1] "On the Constitution of the Interior of the Earth as Revealed by Earthquakes," *Quar. Jour. Geol. Soc.*, LXII (1906), p. 470.

[2] P. 122.

of a thousandth part of that which granite at the surface of the earth can sustain. Furthermore, even if the stresses were greater and could be used as a measure of strength, this would apply to sudden stresses only and the results obtained from elastic vibrations could not be used safely as a means of determining the strength under long-enduring stresses. Thus the evidence from both tides and earthquakes is negative in regard to the existence of an asthenosphere. They show only that it is not fluid and that it is not markedly unlike the rest of the earth in its elastic properties.

HIGH, BUT VARIABLE, ELASTIC LIMIT WITHIN THE UPPER LITHOSPHERE

The experiments by F. D. Adams showed that under conditions of cubic compression rocks became far stronger than when subjected to compression, as at the surface of the earth, in one direction only. When a cylinder of Westerly granite was incased in a steel jacket and then subjected to heavy pressure upon its ends, a small cavity within the specimen just began to break down under a stress-difference of between 160,000 and 200,000 pounds per square inch, about six to eight times the strength possessed by this rock under surface conditions. At a temperature of 550° C., a temperature calculated to exist at a depth of 11 miles below the earth's surface, small cavities remained open when submitted to considerably greater pressures than occur from the overlying load at this depth.[1]

Adams' experiments and King's calculations are most important and show without doubt that the more superficial parts of the earth, to a depth of ten to fifteen miles at least, are far stronger than had been supposed; but they apply to the temperature and pressure gradients in places of geologic quiet, not to regions undergoing igneous intrusion and crustal deformation. Then the temperatures may become far higher and the crust surcharged with magmatic gases. Yet it is under these conditions especially, of geologic activity as contrasted to geologic quiet, that regional metamorphism and rock flowage proceeds. Still less does this experimental work prove a great strength of the crust at depths of more

[1] Louis Vessot King, "On the Limiting Strength of Rocks under Conditions of Stress Existing in the Earth's Interior," *Jour. Geol.*, XX (1912), 136, 137.

than a hundred kilometers, for there the temperatures are presumably above those which under the conditions of freedom from pressure at the surface of the earth produce dry fusion. Occluded gases, furthermore, are held beyond possibility of escape.

The strength of the crust is dependent consequently upon fourfold conditions—the nature of the material, the cubic compression, the relation of temperature to the point of fusion, and the rapidity of the application of the stress. These factors are all variable with time and place. How variable will be seen upon further consideration in the following paragraphs.

The influence of the nature of the material is seen when it is noted that granite is only about one-half as rigid as the basic rocks, although it is not less strong. Consequently, regional stress coming upon a complex of two such rocks will elastically deform the granite more readily, a greater stress will be thrown upon the basic rocks, and since their elastic limit is not correspondingly higher they should begin to yield by flow or fracture before the more pliant rocks had reached their limit. The general conclusion is that a movement of compression in the earth's crust must necessarily give rise to unequal strains and concentration of stress, as well from variations in chemical composition as from variations in structure. The local stress may rise far higher than the general regional stress.

As to the second factor, during the progress of normal faulting the horizontal compressive stress in the crust is less than the vertical stress due to weight. During the progress of folding and mashing, on the contrary, the horizontal stresses become far higher. But the least of the three principal stresses determines the amount of cubic compression; the difference between the greatest and least stresses determines, on the contrary, the amount and direction of the strain upon the rigidity of the rock. Thus it is seen that both the cubic compression and the stress-difference vary with the amount and kind of forces.

It is temperature, however, which is probably the most variable of these factors. Igneous activity brings the temperatures of the greater depths comparatively near to the surface and must produce

widespread weakening of the crust, both through the physico-chemical effects of the exalted temperatures and the structural effects of the intruded viscous fluids.

The rapidity of the application of stress is a variable in itself and furthermore has variable effects, but would seem, however, to be the least important of these several factors. The movements of horizontal compression and vertical warping are slow and give time for recrystallization in the deeper crust. In this way they meet a lesser resistance than would rapid stresses. Where the temperatures are close to those of fusion it would seem in fact that rock flowage by recrystallization, developing the gneissoid structure, should demand markedly less shearing stress than the process of granulation. The gnarled and twisted rocks of the Archean speak of the presence beneath them of molten magmas rather than of an enormous degree of compressive forces upon them. But ready yielding by recrystallization in one place would permit the concentration of mashing stresses upon other localities and raise the strain to that intensity needed for granulation. An enormous depth of cover, such as Adams' experiments have been thought to show, is not suggested by the geologic evidence, nor apparently is it demanded by a completer theory.

In fault movements and in dike or sheet intrusion accompanied by the expansion of gases are two sources of rapid application of forces. It is probable, however, that their deformative action is confined to the outer ten miles of the crust, and their consideration need not detain us in the evaluation of those factors of strength which concern the crust as a whole.

Summing up the conclusions from these various lines of evidence, physical and geological, it is seen that they suggest a rapid increase of strength with depth, then the gradual passage into a deep zone of lowered strength. The limits and values, however, are variable with time and place. Such a distribution of strength as is indicated by these independent lines is in accord with the interpretation of the geodetic evidence showing the existence of crustal competence to support heavy loads over certain limits of area, coexisting with flotational equilibrium over much broader regions.

MODES OF LITHOSPHERIC YIELDING AND THEIR RELATION TO STRENGTH

The relationship of strength to depth which has been derived in this study and which was expressed in the curve of strength at the end of Part VII is to be connected with the physical qualities discussed in this part. Here it is seen that it is a curve of elastic limit. When that limit is exceeded, permanent deformation must take place; by one means at the surface, by another within the body of the lithosphere, by still another at its base.

At the surface the typical mode of yielding is by jointing and faulting, in stratified beds by folding also. The movements in this zone of fracture and in the transitional zone of combined fracture and flow may be regarded as merely the responses in a thin, brittle, and relatively weak outer layer to deformative movements progressing in the great thickness of the lithosphere below. But the rocks of deeper origin which have been exposed at the surface by profound erosion show that they have yielded in another fashion. Their foliated structures and crystalline textures testify to yielding by massive flowage. Fracturing appears to have been absent, except in so far as it was produced by intrusions from below, giving rise to complexes of dikes and sheets. These visible exposures suggest that at still greater depths, notwithstanding the great strength of that zone, open fracture planes disappear and rock flowage both by granulation and by recrystallization is still more distinctive. This appears then to be the mode of yielding of the great body of the lithosphere.

Recently Becker has suggested that fracturing may enter into the problem of isostasy in the following way: The demonstrated capacity of small cavities to remain open under great pressures may permit fissuring and jointing to extend deeper into the crust than had been previously thought possible. To the degree to which fractures and porosities do exist they must decrease the specific gravity of rocks. If shattering pervaded the rocks of one region and not another, even though the rocks were exactly alike in composition, the densities would become different. To give isostatic equilibrium the region of shattered rocks would have to stand higher than the other. This would be the initial effect as

a result of the decrease in density, even if the zone of compensation rested on an unyielding base.[1] The logical correctness of this argument is not to be questioned, but rather the degree of its application. The following arguments suggest that shattering or porosity are, however, very subordinate rather than determining factors in the isostatic problem.

Such a theory does not account readily for the movements needed to maintain isostasy because of erosion and sedimentation. These surface changes of mass suggest a restoration of mass by lateral undertow. Furthermore, the appeal to nature shows that the rocks, once deep-seated, which have become revealed at the surface by erosion, are almost without pore space. The average porosity according to Fuller is 0.2 per cent, but the mean differences in densities between ocean and continent which must be accounted for under the hypothesis of uniform compensation to a depth of 122 km. amount to about 4 per cent.

Joints are observed to decrease with depth, becoming tighter and more distantly spaced, and the indications given by the lack of general circulation of ground-water through crystalline rocks, except within joint spaces near the surface, are that at greater depth the joint spaces are negligible.

In the great compressive movements the whole thickness of the crust must yield, but even this cannot be conceived as producing porosity by granulation sufficient to notably modify the density. A large part of the deformation in the deeper crust must be by a process of recrystallization. Assume, however, that granulation is the dominant process. Observation of granulated rocks shows a reduction in size of the crystals, but these broken fragments fit against each other perfectly and without great internal distortion of crystals. In granulated rocks from the zone of flow there is therefore always some amount of recrystallization, sufficient to eliminate that porosity connected with minute shattering and movement of the broken particles. The explanation appears to be as follows: The minute shattering of the minerals tends to give a high pore space, but with a high pore space the amount of contact

[1] G. T. Becker, "Isostasy and Radioactivity," *Science*, XLI (1915), 157–60; "On the Earth Considered as a Heat Engine," *Proc. Nat. Acad. Sci.*, I (1915), 81–86.

between grains becomes proportionately less. For the prevention of ready recrystallization and the maintenance of this pore space the granulated rock, according to present theory, must be conceived of as dry and the grains accordingly unsupported except at the points of contact. The shear strains within each grain become very great in proportion to the diminution of contact, and increase in proportion to the regional pressure. If the points of contact, for example, cover only one-fourth of the surface, the compression on those points would be four times as great per unit of surface as if there were continuous contact between grains. On the intervening parts of the surface there would be no pressure. Internal shears would result in this way from the hydrostatic pressure of dry rock due to depth and are not dependent upon a pressure-difference in the rock as a whole. The internal strains would tend to produce molecular changes of state as in the plastic flow of metals. There would be melting to relieve the strain, and refreezing by which the molecules would build out the crystals into the pore spaces. By this means recrystallization can go on without the aid of crystallizers, though presumably with more difficulty, and the comminuted crystals come to fit compactly as they are observed to do. This elimination of porosity presumably goes on approximately with the process of granulation, though it may lag somewhat. It would go forward more effectively with depth, irrespective of temperature, since there would be the greater static load upon the rock and the greater differential pressures within the mineral particles. It might be expected that such reduction of pore space would go forward to a limited extent only, leaving a residual porosity. Observation, however, shows that the pore space has been almost completely eliminated. Furthermore, the rocks now exposed at the surface acquired their absence of pore space at depths of only a few miles from the surface. At depths measured in tens of miles there seems then no expectation that density would be notably decreased because of a development of porosity.

To sum up the modes of yielding within the lithosphere: at the surface is seen to exist a thin outer crust intimately cracked on the outside by closely spaced parallel joint systems. Local

extreme deformation is by faults and folds. With increasing depth and strength the joints become less abundant and faults pass into flexures. The passage of fractures into flexures implies the beginnings of massive flow. Where magmatic heat or emanations are not present the mode of mashing is presumably more especially by granulation. With still greater depth the yielding becomes more uniformly distributed throughout the rock mass. Both because of this pervasiveness of mashing and the great strength of this zone, deformation here requires the most force and absorbs the most energy of any part of the lithosphere. At greater depths the rock is more compressed, and is still more rigid than above, but the temperature here approaches fusion; recrystallization readily takes place, the strain which can be elastically carried is in consequence low, and the lithosphere passes gradually into the asthenosphere. Where, however, magmas rise through the crust they carry with them the environment of the asthenosphere; the lithosphere becomes locally abnormally heated and saturated with magmatic emanations. Recrystallization goes forward readily and the zone of weakness penetrates upward even to the zone of fracture. Thus in the injected and crystallized roofs of ancient batholiths, laid bare by profound erosion, we may perceive the nearest approach to dynamic conditions which prevail in depths forever hidden.

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